THEORY & OBJECTIVE

FLUID MECHANICS

By
Team of
Engineers Academy

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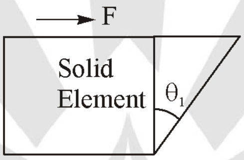
INTRODUCTION

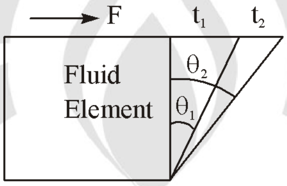
THEORY

1.1 FLUID

The substances which continuously deform under the application of a tangential or shear stress no matter how small is the value of tangential force is known as fluid.



θ is the deformation at time t_1 and t_2 if force(F) is constant



θ_1 at time t_1; θ_2 at time t_2 if $t_2 > t_1$ then $\theta_2 > \theta_1$ for fluid element

where,

θ_1 = Deformation at time t_1

θ_2 = Deformation at time t_2

1.2 PROPERTIES OF FLUID

1.2.1 Density or Mass Density

Mass density is define as the mass of the system corresponding to its volume.

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

$$\rho = \frac{m}{\forall}$$

$$\text{Unit} = \left(\frac{kg}{m^3}\right)$$

$$\text{Dimensional formula} = \left[M^1L^{-3}T^0\right]$$

For ideal gases

$$P\forall = mRT$$

$$P = \frac{m}{\forall}RT$$

$$P = \rho RT$$

where, R is the gas constant

$$R = 0.287 \frac{kJ}{kg.k} \text{ for air.}$$

Note:

a) Density is absolute quantity.

b) Density of gas is dependent on the temperature and pressure of the system.

1.2.2 Specific Gravity (S)

$$S = \frac{\text{density of substance}}{\text{density of standard fluid}}$$

Note:

a) For liquid standard fluid is water (1000 kg/m^3)

b) For gases standard fluid is either air or hydrogen gas.

c) Specific gravity of Hg is 13.6

1.2.3 Relative Density (R.D.)

$$\text{Relative density (R.D.)} = \frac{\text{density of one fluid}}{\text{density of another fluid}}$$

$$\text{R.D.} = \frac{\rho_1}{\rho_2}$$

1.2.4 Specific Weight or Weight Density (w or γ):

It is define as the ratio of weight of system to the volume of the system.

$$w = \frac{\text{weight}}{\text{volume}} = \frac{\rho g \forall}{\forall} = \rho g$$

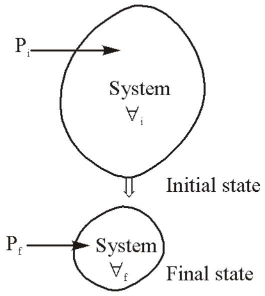
$$\text{Unit} \quad \left(\frac{kg}{m^3} \times \frac{m}{sec^2}\right) = \left(\frac{kg}{m^2.sec^2}\right)$$

$$\text{Dimensional formula} = \left[M^1L^{-2}T^{-2}\right]$$

Note:

Specific weight is the relative quantity and it depends on the pressure, temperature and location.

1.2.5 Bulk Modulus and Compressibility



If original pressure is P in a liquid mass and its volume is $\forall$ and increase in pressure dP causes change in volume d$\forall$ then

$$dP = P_f - P_i$$

$$d\forall = \forall_f - \forall_i = -(\forall_i - \forall_f)$$

$$\text{Bulk modulus of elasticity (K)} = -\frac{dP}{(d\forall/\forall)} \qquad \left[\text{analogous to } \frac{\text{stress}}{\text{strain}} = \text{Young's modulus of elasticity}\right]$$

Unit of K will same as pressure (Pa, N/m^2)

$$\boxed{K = -\frac{dP}{\left(\frac{d\forall}{\forall}\right)} = \frac{dP}{\left(\frac{d\rho}{\rho}\right)}}$$

$$\rho\forall = \text{mass} = \text{constant}$$

$$\rho d\forall + \forall d\rho = 0$$

$$\Rightarrow \quad \frac{d\rho}{\rho} = -\frac{dV}{\forall}$$

$$\boxed{\text{Compressibility} = \frac{1}{K}} = \frac{1}{\rho}\frac{d\rho}{dP}$$

Note : If density does not change with pressure i.e., $\frac{d\rho}{dP} = 0$ (fluid is incompressible with respect to pressure).

1.2.6 Isothermal Bulk Modulus (K_T)

For ideal gas $\quad P = \rho RT$

$$R = 0.287\frac{kJ}{kg-k}$$

$$\Rightarrow \quad \left(\frac{dP}{d\rho}\right)_{T=\text{Constant}} = RT$$

$$\Rightarrow \quad K_T = \rho\frac{dP}{d\rho} = \rho \cdot RT = P$$

⇒ for isothermal condition in an ideal gas,

$$K_T = P$$

i.e, $\boxed{\text{Isothermal bulk modulus = pressure}}$

1.2.7 Adiabatic Bulk Modulus (K_A)

For adiabatic condition $\quad P\forall^{\gamma} = \text{constant}$

where, $\quad \gamma = \text{adiabatic index}$

$$= \frac{Cp}{Cv} = \frac{\text{Specific heat at constant pressure}}{\text{Specific heat at constant volume}}$$

$$\Rightarrow \quad P\left(\frac{m}{\rho}\right)^{\gamma} = \text{constant}$$

$$\frac{P}{\rho^{\gamma}} = \text{constant (because mass ‘m’ is constant)}$$

$$\Rightarrow \quad \boxed{P = Cp^{\gamma}}$$

$$\frac{dP}{d\rho} = \rho C\rho^{(\gamma-1)}$$

$$K_A = \rho\frac{dP}{d\rho} = \gamma C\rho^{\gamma} = \gamma P$$

$$\Rightarrow \quad \boxed{\text{Adiabatic bulk modulus} = \gamma P = \text{Adiabatic index} \times \text{Pressure}}$$

Example 1 : The bulk modulus of water is 210 × 10^4 kN/m^2. What pressure is required to reduce the volume of water in a closed container by 2%; Also show that the increase in mass density because of the reduction in volume by 2% will be 2% only.

Solution : Given

$$k = 210 \times 10^4 \text{ kN/m}^2$$

$$\frac{d\forall}{\forall} = 0.02$$

From the definition of bulk modulus of elasticity

$$k = -\frac{dp}{\left(\frac{d\forall}{\forall}\right)} \qquad \text{...(1)}$$

For 2% reduction in volume

$$\left(\frac{d\forall}{\forall}\right) = 0.02 \qquad \text{...(2)}$$

$$\therefore \quad 210 \times 10^4 \text{ kN/m}^2 = \frac{dP}{(0.02)}$$

$$\Rightarrow \quad dP = 210 \times 10^4 \times 0.02 \text{ kN/m}^2$$

$$\therefore \quad dP = 42{,}000 \text{ kN/m}^2$$

$$dP = 42 \text{ MN/m}^2$$

Thus additional pressure required to reduce the volume in closed container by 2% is 42 MN/m^2

∴ For constant mass in the closed container,

$$\frac{d\rho}{\rho} = -\left(\frac{dV}{V}\right)$$

$$= -(-2\%) = 2\%$$

Thus reduction in volume by 2% will lead to increase in mass density by 2%.

Example 2 : An empirical pressure-density relation for water in $\frac{P}{P_a} = 3001\left(\frac{\rho}{\rho_a}\right)^7 - 3000$ where suffix 'a' refers to atmospheric conditions. Determine the bulk modulus of elasticity and compressibility of water at 5 atmospheric pressure.

Solution :

$$\frac{P}{P_a} = 3001\left(\frac{\rho}{\rho_a}\right)^7 - 3000 \qquad \text{...(1)}$$

differentiating both side.

$$\frac{dP}{P_a} = \frac{3001 \times 7.\rho^6 d\rho}{\rho_a^7} \Rightarrow \frac{dP}{d\rho} = \frac{3001 \times 7 \times \rho^6 \times P_a}{\rho_a^7} \qquad \text{...(2)}$$

We know that,

$$K = \frac{-dP}{\left(\frac{dv}{v}\right)} = \frac{dP}{\left(\frac{d\rho}{\rho}\right)} = \left(\frac{\rho dP}{d\rho}\right) \qquad \text{...(3)}$$

Put the value of $\frac{dP}{d\rho}$ from (2) in (3)

$$\therefore \quad K = \frac{3001 \times 7 \times \rho^6 \cdot \rho}{\rho_a^7} P_a$$

$$\therefore \quad K = 3001 \times 7 \left(\frac{\rho}{\rho_a}\right)^7 \times P_a \qquad \text{...(4)}$$

Put the value of (1) in (4)

$$K = 3001 \times 7 \frac{\left(\frac{P}{P_a} + 3000\right)}{3001} \times P_a$$

$$K = 3001 \times 7 \frac{[5 + 3000]}{3001} \times P_a$$

$$= 7 \times 101.325 \times 3005 \qquad (\text{where } P_{atm} = 101.325 \text{ kN/m}^2)$$

$$= 2.131 \times 10^6 \text{ KN/m}^2$$

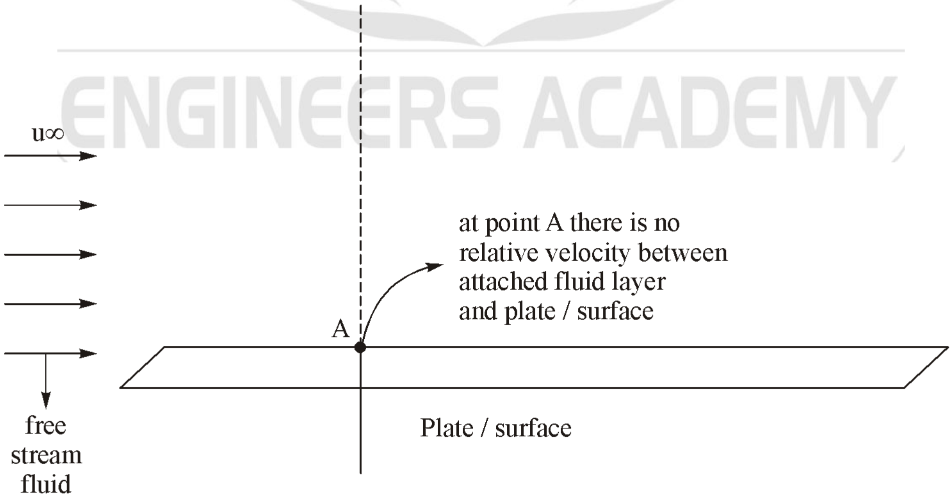
$$\therefore \quad \text{The compressibility} = \left(\frac{1}{K}\right) = \frac{1}{(2.131 \times 10^6)} = 4.693 \times 10^{-7} \text{ m}^2/\text{KN}$$

1.3 NO SLIP CONDITION

It has been established that the relative velocity between the solid surface and the adjacent fluid particle is zero when ever a viscous fluid flows over a solid surface. This is known as no-slip condition. This behavior of no-slip at the solid surface is not the same as the wetting of surface by the fluids. For example mercury flowing in a stationary glass tube will not wet the surface, but will have zero velocity at the wall of the tube.

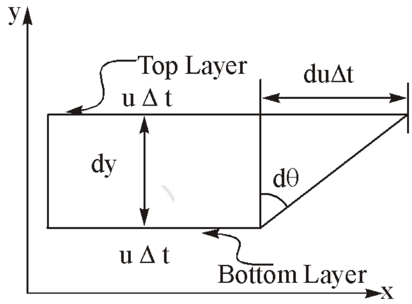
The wetting property results from surface tension, whereas the no-slip condition is a consequence of fluid viscosity.

In ideal fluid as there is no viscosity hence there will be no no-slip condition existing.



1.4 DYNAMIC VISCOSITY OR ABSOLUTE VISCOSITY

Dynamic viscosity is define as the property of the fluid which develop the internal resistance between two adjacent layer of fluid when they have a relative motion.



$$\tan\theta = \frac{du\,\Delta t}{dy}$$

if, $\Delta t \to 0$

$$\tan d\theta \approx d\theta$$

$$d\theta = \frac{du\,dt}{dy}$$

$$\frac{d\theta}{dt} = \frac{du}{dy}$$

Rate of change of shear strain $\left(\frac{d\theta}{dt}\right)$ = velocity gradient $\left(\frac{du}{dy}\right)$

1.5 NEWTON'S LAW OF VISCOSITY

According to Newton's Law of viscosity shear stress is directly proportional to the shear strain rate or velocity gradient in transverse direction.

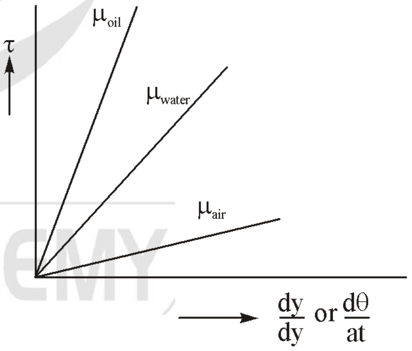
$$\tau \propto \frac{d\theta}{dt}$$

$$\tau = \mu\frac{d\theta}{dt} = \mu\frac{du}{dy}$$

where, μ is the dynamic viscosity of fluid.

Note : A fluid which obey Newton's Law of viscosity is known as Newtonian fluid.

1.5.1 Units of Dynamic Viscosity



(i) Pa.sec in SI unit

(ii) $\frac{kg}{m.sec}$ in M.K.S. unit

(iii) 1 poise = $10^{-1}\frac{kg}{m.sec}$ in CGS

Dimensional formula = $\left[M^1L^{-1}T^{-1}\right]$

1.5.2 Effect of Temperature on Viscosity of Fluid

a) In case of liquid, cohesive forces between molecules cause viscosity. Hence at higher temperature when molecules possesses more energy, cohesive force become weak. As a result, energized liquid molecule can move more freely, thus viscosity of liquid decreases with increase in temperature.

$$\boxed{\mu_{liq} = \frac{\mu_0}{1+\alpha t+\beta t^2}}$$

α and β are constants,

t = temperature in °C.

μ_0 = viscosity of liquid at 0°C

For water $\mu_0 = 1.79 \times 10^{-3}$ poise

$\alpha = 0.03368$

$\beta = 0.000221$

b) In case of gases, viscosity is caused by molecular collisions. At high temperature, more molecular collisions per unit volume per unit time occurs. This results in greater resistance to flow. Thus,

Viscosity increases with temperature

$$\mu_{gas} = \frac{a\sqrt{T}}{1+\frac{b}{T}}$$

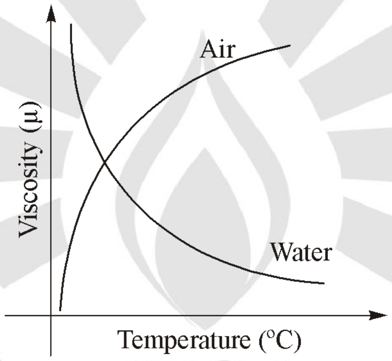
or $$\boxed{\mu_{gas} = \mu_0 + \alpha t - \beta t^2}$$

For Air $\mu_0 = 0.000017$

$\alpha = 0.000000056$

$\beta = 0.1189 \times 10^{-9}$

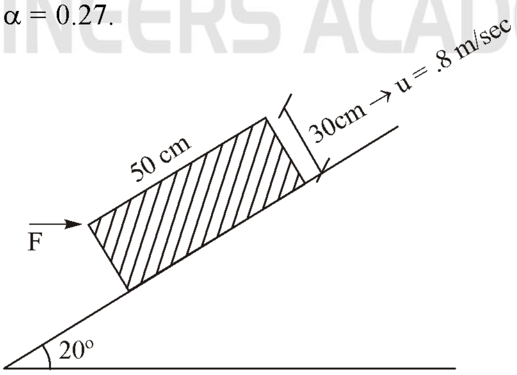
Note : At constant pressure, if temperature is increased 'μ' increases.



1.5.3 *Effect of Pressure on Viscosity*

For liquids, viscosity is practically independent of pressure except at extremely high pressure. For gases, dynamic viscosity is generally independent of pressure particularly (at low to moderate pressure) but kinematic viscosity decreases as density is proportional to pressure.

Example 3 : A 50cm × 30cm × 20cm block weighing 150N is to be moved upon inclined plane at constant velocity of 0.8 m/sec coefficient of friction α = 0.27.



(a) Determine the force F required to be applied in horizontal direction to maintain this motion.

(b) If a 0.4 mm thick oil film with dynamic viscosity μ = 0.012 Pa.sec is applied between the block and inclined surface determine the percentage reduction, in the required horizontal force.

Solution : **(a)**

$\therefore \quad \Sigma F_y = 0 \Rightarrow$ because of constant volume $\Rightarrow$ (acceleration = 0)

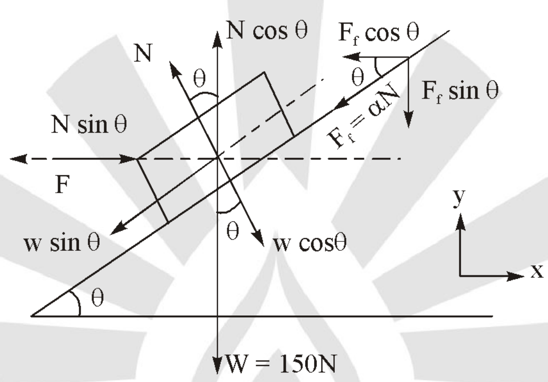
$$\therefore \quad N\cos\theta - W - F_f \sin\theta = 0$$

$$N\cos\theta - W - \alpha N \sin\theta = 0$$

$$\because \frac{F_f}{N} = \alpha = \tan\theta \qquad \text{...(i)}$$

$$\Rightarrow \quad N(\cos\theta - \alpha\sin\theta) = W \quad F_f = N\cdot\alpha$$

$$\therefore \quad N = \frac{W}{(\cos\theta - \alpha\sin\theta)}$$



$$= \frac{150}{(\cos 20^\circ - 0.27\sin 20^\circ)} = 177.023 \qquad \text{...(ii)}$$

$$\Sigma F_x = 0$$

$$\therefore \quad F - N\sin\theta - F_f\cos\theta = 0 \qquad \text{...(iii)}$$

$$F = N\sin\theta + \alpha N\cos\theta$$

$$= N(\sin\theta + \alpha\cos\theta)$$

$$F = 177.023(\sin 20^\circ + 0.27\cos 20^\circ)$$

$$F = 105.5\text{ N} \quad \text{...(iv)}$$

(b) $\therefore \quad \left(\frac{du}{dy}\right) = \left(\frac{u}{d}\right)$

$$\therefore \quad \tau = \mu\left(\frac{du}{dy}\right) = \left(\frac{\mu u}{d}\right)$$

$\therefore \quad F_{shear} =$ (Shear stress × Area of contact)

$$= 0.012 \times \left(\frac{0.8}{0.4\times 10^{-3}}\right) \times 50 \times 20 \times 10^{-4}$$

$$= 2.4\text{ N} \qquad \text{...(v)}$$

Applying N.S.L. along the plane

$$F_{net} = m \times a$$

$$F\cos 20^\circ - W\sin 20^\circ - \tau \times A = m \times 0$$

$$F\cos 20^\circ = W\sin 20^\circ + \frac{\mu V}{h}.A$$

$$F\cos 20^\circ = 150\sin 20^\circ + \frac{0.012 \times 0.8 \times 50 \times 20 \times 10^{-4}}{0.4 \times 10^{-3}}$$

$$F\cos 20^\circ = 53.70$$

$$F = 57.15\text{ N}$$

$$\therefore \quad \text{Percent reduction} = \left(\frac{105.5 - 57.149}{105.5}\right) \times 100$$

$$= 45.829\%$$

Example 4 : A thin square plate 1m × 1m is placed horizontally in a gap of height 2cm. Filled with oil of viscosity 10 Poise and pulled at a constant velocity of 0.10 m/sec. Find the force on the plate.

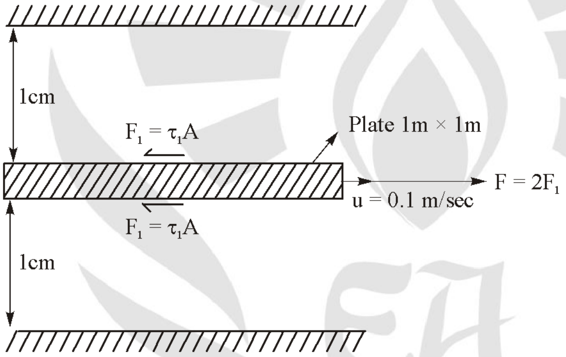
The gap is now filled with another oil, when the plate is placed at a distance of 0.5cm, from one of the surfaces of the gap and pulled with the same velocity, the force on the plate remains same as before. Find viscosity of new oil.

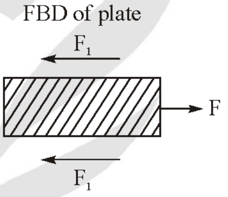
Solution : Data given:

$$\mu = 10\text{ Poise} = 1.0\frac{\text{N.s}}{\text{m}^2}$$

$$u = 0.1\text{ m/sec.}$$

Case-I :





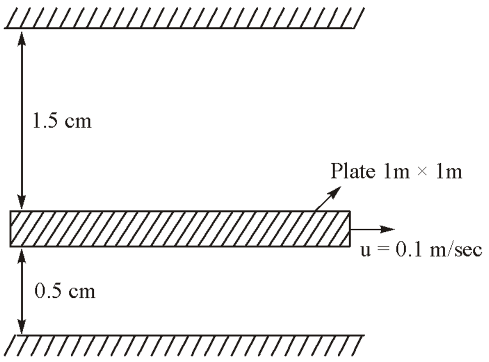
Force on one side of plate

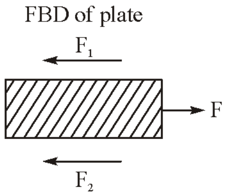
$$\Sigma F = m\vec{a} \quad (\vec{a} = 0)$$

$$F_1 = \mu\left(\frac{du}{dy}\right) \times A = 1 \times \frac{0.1}{(1 \times 10^{-2})} \times 1 = 10\text{N}$$

$$\therefore \quad F = 2F_1 = 2 \times 10 = 20\text{ Newton} \qquad \text{...(1)}$$

Case-II :





Force on top of plate = $\mu\left(\frac{dy}{dy}\right)_{top} \times \text{Area}$

$$=\mu\frac{0.1}{1.5\times10^{-2}}\times1=\frac{20}{3}\mu \qquad ...(2)$$

Force on bottom of plate

$$= \mu\left(\frac{du}{dy}\right)_{bottom}\times\text{Area}=\mu\frac{0.1}{\left(0.5\times10^{-2}\right)}\times1=20\mu$$

Total force in Case (2) $= \left(\frac{20}{3}\mu+20\mu\right)=\left(\frac{80}{3}\mu\right)$ ..(3)

From (1) and (3)

$$\frac{80}{3}\mu = 20$$

$$\therefore \qquad \mu = \left(\frac{3}{4}\right)=0.75\left(\frac{N.s}{m^2}\right)$$ **Ans.**

Example 5 : Through a narrow gap of height h, a thin plate of large extent is pulled at a velocity V, at one side of the plate. Oil of viscosity μ_1, and on other side μ_2. Calculate the position of plate so that

(a) Shear force on two sides of the plate are equal.

(b) The pull required to drag to plate is minimum.

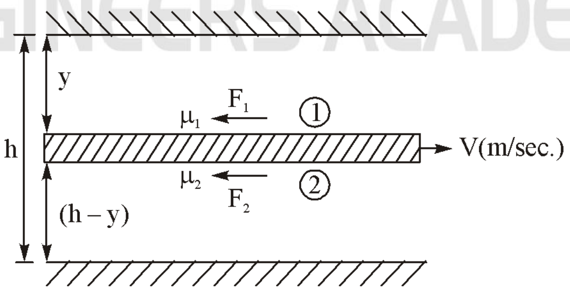
Solution :(a) ∴ Shear force on side (1) of plate

$$\Sigma F = ma\,(a=0)$$

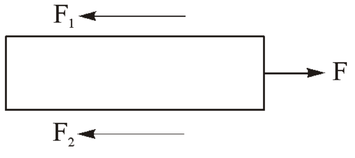
$$F_1 = \tau_1 A = \left(\frac{\mu_1 AV}{y}\right) \qquad ...(1)$$

Shear force on other side of plate

$$F_2 = \tau_2 A = \left(\frac{\mu_2 AV}{h-y}\right) \qquad ...(2)$$



FBD of plate



(a) From (1) and (2)

$$\frac{\mu_1 AV}{y} = \frac{\mu_2 AV}{(h-y)}$$

$$\Rightarrow \quad \mu_1 h - \mu_1 y = \mu_2 y$$

$$\Rightarrow \quad \mu_1 h = (\mu_1 + \mu_2) y$$

$$\therefore \quad y = \frac{\mu_1 h}{(\mu_1 + \mu_2)} \qquad \text{...(3)}$$

(b) Pull required to drag the plate

$$\therefore \text{ total drag force} \quad F_D = \frac{\mu_1 AV}{y} + \left(\frac{\mu_2 VA}{h-y}\right)$$

For F_D to be minimum.

$$\left(\frac{dF_D}{dy}\right) = 0 \qquad \text{For minima of maxima.}$$

$$\therefore \quad \frac{dF_D}{dy} = -\frac{\mu_1 VA}{y^2} + \frac{\mu_2 AV}{(h-y)^2} = 0$$

$$\therefore \quad \frac{\mu_2}{(h-y)^2} = \frac{\mu_1}{y^2}$$

$$\Rightarrow \quad \left(\frac{\mu_2}{\mu_1}\right) = \left(\frac{h-y}{y}\right)^2 \qquad \text{(where } \mu_1 \text{ and } \mu_2 \text{ are dynamic viscosities)}$$

$$\therefore \quad \frac{h-y}{y} = \sqrt{\left(\frac{\mu_2}{\mu_1}\right)}$$

$$\Rightarrow \quad h\sqrt{\mu_1} - y\sqrt{\mu_1} = \sqrt{\mu_2}\, y$$

$$\Rightarrow \quad h\sqrt{\mu_1} = y\left(\sqrt{\mu_1} + \sqrt{\mu_1}\right)$$

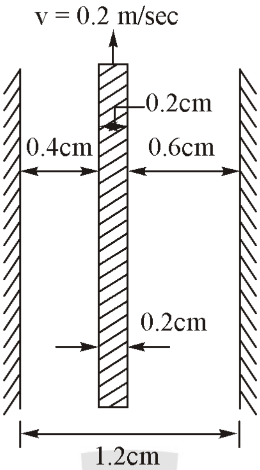
$$\Rightarrow \quad y = \frac{h\sqrt{\mu_1}}{\left(\sqrt{\mu_1} + \sqrt{\mu_2}\right)}$$

Example 6 : A horizontal gap 1.2 cm. Wide of infinite extent contains fluid of viscosity 1 N.S/m^2 and specific gravity 0.9. A metallic plate 1m × 1m × 0.2 cm is lifted up with a constant velocity of 0.2 m/sec through the gap. If the plate is at a distance of 0.4 cm, from one of the plane surface of the gap, find the vertical force required. Weight of plate is 50N.

Solution : As the given plate is submerged in the fluid of specific gravity (0.9)

Total forces acting on the plate are (a) Submerged weight in downward direction of plate (b) Drag force so total force required to life the plate

= Submerged wt. of plate + total drag on the plate



Submerged wt. of the plate $= \text{wt} - (\text{sp. gravity}) \times \gamma_w \times \text{vol. of plate}$

$$= 50 - 0.9 \times 9.81 \times 10^3 \times \left(1 \times 1 \times 0.2 \times 10^{-2}\right)$$

$$= 32.342\,\text{N}$$

and total drag force on the plate

$$\Rightarrow \qquad F_D = \mu \cdot \left(\frac{du}{dy}\right) \times (\text{Area})$$

$$= \left\{1 \cdot \left(\frac{0.2}{0.4 \times 10^{-2}}\right) \times 1 \times 1\right\} + \left\{1 \cdot \frac{0.2}{0.6 \times 10^{-2}} \times 1 \times 1\right\}$$

$$= 50 + \frac{100}{3} = 83.33\,\text{N}$$

Hence total force required to lift the with constant velocity

$$= 32.342 + 83.33$$

$$= 115.672\,\text{N}.$$

1.6 RHEOLOGY

It is a branch of science in which we study different types of non-newtonian fluids on the basis of their behavior (flow behavior)

1.6.1 Non Newtonian Fluid

A fluid which do not obey Newton's law of viscosity is known as Non-Newtonian fluid. It obey power law relation

$$\tau = A\left(\frac{du}{dy}\right)^n + B$$

where $A =$ consistency index

$n =$ flow behavior index

or

$$\tau = A\underbrace{\left(\frac{du}{dy}\right)^{n-1}}_{(\mu \text{ aparent})}\left(\frac{du}{dy}\right)$$

a) Pseudo Plastic Fluid :

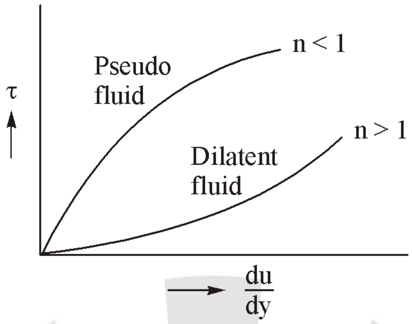
For $B = 0$ and $n < 1$ and fluid is also known as shear thinning fluid.

Example: Milk, blood, ketch up

b) Dilatant Fluids (Shear Thickening Fluid) :

$B = 0; n > 1$; Dilatant fluid is known as shear thicking fluid.

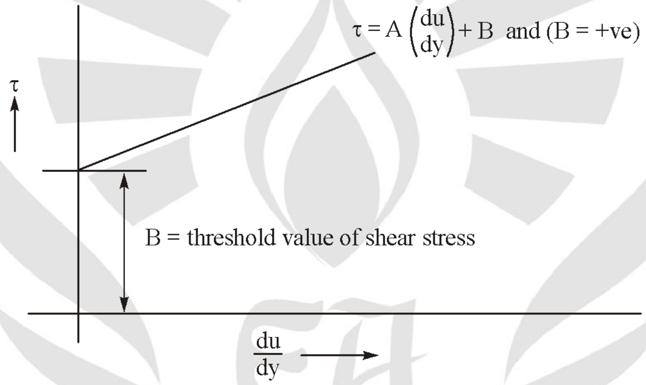
Example: honey, saturated solution of sugar, rice-starch solution.



c) Bingham Plastic Fluids (Ideal Plastic Fluid):

$$n = 1 ; B \neq 0$$

$$\tau = A\left(\frac{du}{dy}\right) + B \text{ and } (B = +ve)$$

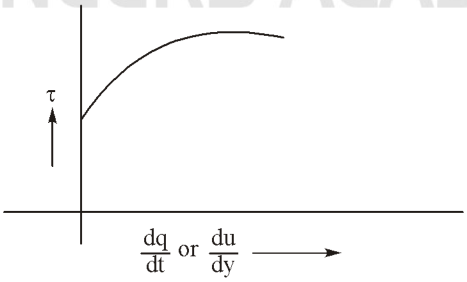


Example: Toothpaste, hair gel, facewash cream and jelly.

d) Thixotropic Fluid :

Thixotropic fluid is defined as the apparent viscosity increases with respect to time.

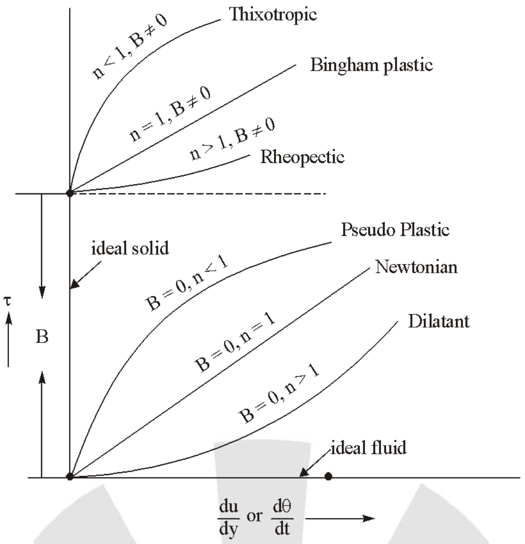
Example: Drilling water, castrol oil and antibiotic.



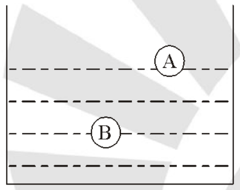
e) Rheopectic Fluid :

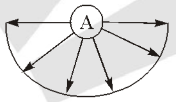
Apparent viscosity decreases with respect to time, such fluid is known as rheopectic fluid.

Example: Gypsum in water and Printer ink.

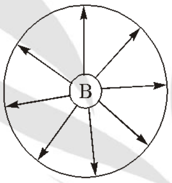


1.7 SURFACE TENSION

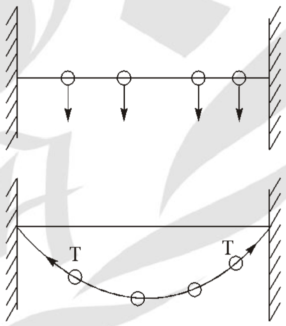




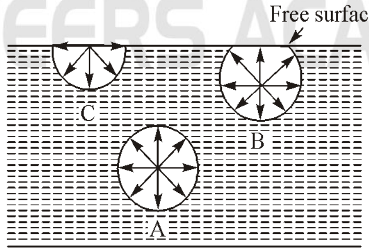
Hemispherical force
cohesive force ≠ 0



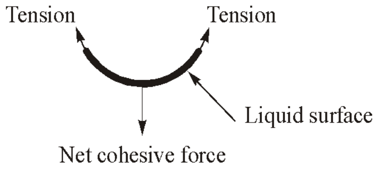
Hemispherical force
$F_{cohesive} = 0$



$F_{accumulative} = F_{cohesive}$



Net down force on c is shown here



There is a tensile force acting on the surface. As a result of net downward force, surface will be pulled down causing a curvature to the surface. This in turn develops a tension in the surface which is similar to the tension in a stretched membrane. This is known as surface tension. It is a internal force.

It is the force per unit length with unit of N/m or J/m^2

Dimensional formula : $[M^1L^0T^{-2}]$

$$\sigma_{water} = 0.073\frac{N}{m} \quad \text{at } 20°C$$

$$\sigma_{Hg} = 4.56\frac{N}{m}$$

For same volume, sphere has minimum surface area.

It is a property of liquid by virtue of which it minimizes or tries to minimum its surface area is known as surface tension.

$$\sigma = \frac{F_T}{\ell}$$

Where $F_T \rightarrow$ Tangential force

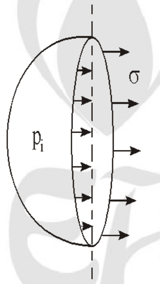
$$\sigma = \frac{F_T}{\ell} \times \frac{\Delta x}{\Delta x}$$

$$\sigma = \frac{\text{work done}}{\text{change in surface area}}$$

So surface tension is also define as a required work done to change in unit surface area.

1.7.1 Application of Surface Tension

(a) Water Droplet :



If R is the radius of drop.

Surface tension force = pressure force

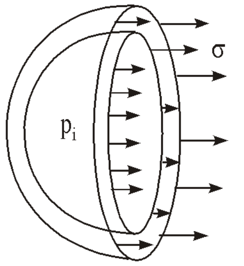
Then $F_S = F_P$

$$\sigma.2\pi R = \Delta P\,\pi R^2 \qquad (\Delta P = P_i \sim P_o)$$

$$\Delta P = \left(\frac{2\sigma}{R}\right)$$

$$\Delta P = \frac{4\sigma}{D} \qquad (\text{where } D = 2R)$$

(b) Soap Bubble :



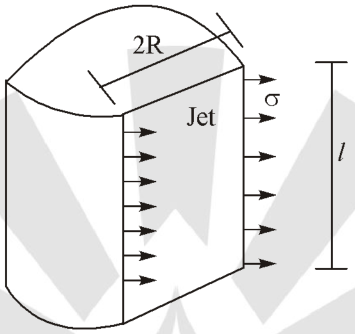
By force balancing in horizontal direction.

$$\sigma \cdot 2\pi R + \sigma \cdot 2\pi R = \Delta P\,\pi R^2$$ (in bubble there is two face)

$$\Delta P = \frac{4\sigma}{R}$$

$$\Delta P = \frac{8\sigma}{D}$$

(c) Jet :



Force balancing in horizontal direction

$$\sigma \cdot \ell + \sigma \cdot \ell = \Delta P \cdot 2R\ell$$

$$2\sigma\ell = \Delta P \cdot 2R\ell$$

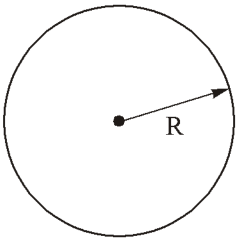
$$\Delta P = \frac{\sigma}{R} = \frac{2\sigma}{D}$$ (where d is the diameter of pipe)

Example 8 : Spherical droplet of radius R is split into n-smaller droplet of equal size.Find the work required? Given that surface tension is equal to σ.

Solution : We know that surface tension is work done per unit increase in surface area.

$$\therefore \quad n \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$$

$$r = \left(\frac{R^3}{n}\right)^{\frac{1}{3}} = \frac{R}{(n)^{\frac{1}{3}}} \quad \text{...(I)}$$



$$\therefore \quad \text{Increase in surface area} = \left(4\pi r^2\right) \times n - 4\pi R^2 = 4\pi\left[nr^2 - R^2\right]$$

$$= 4\pi\left[n\left(\frac{R}{n^{\frac{1}{3}}}\right)^2 - R^2\right]$$

$$= 4\pi R^2\left[n^{(1-2/3)} - 1\right] = 4\pi R^2\left[n^{1/3} - 1\right]$$

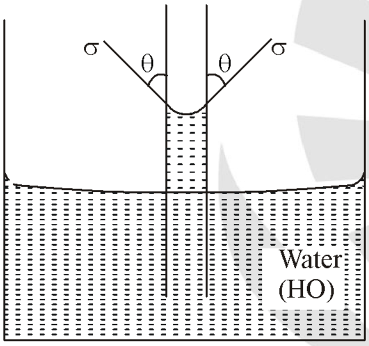
We have, $$\sigma = \frac{\text{work done}}{(\text{Increase in surface area})}$$

$\therefore$ Work done = σ × increase in surface area

$\Rightarrow$ $$W = 4\sigma\pi R^2\left[n^{1/3} - 1\right]$$

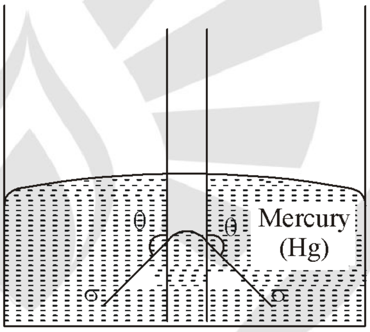
1.8 CAPILLARITY

Capillarity effect is a consequence of cohesion and adhesion. It is defined as the rise or fall of a liquid in a small diameter tube inserted into liquid. The rise is called capillary rise and fall is called capillary depression. The diameter of the tube is less than 1cm.



θ = acute angle
$\theta < 90^\circ$
σ = surface tension

Fig. (1)

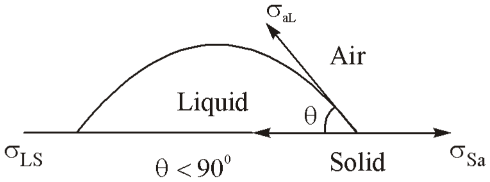


σ = surface tension
$\theta > 90^\circ$
θ is obtuse angle

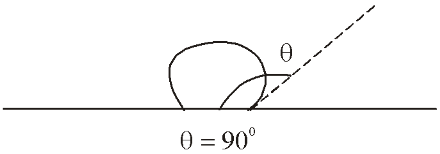
Fig. (2)

1.8.1 Angle of Contact

$\theta < 90^\circ$	cohesion < adhesion	liquid wets solid surface	top surface of water is concave	liquid rises in capillary tube
$\theta > 90^\circ$	cohesion > adhesion	liquid does not wets solid surface	top surface of water is convex	liquid falls in the capillary tube



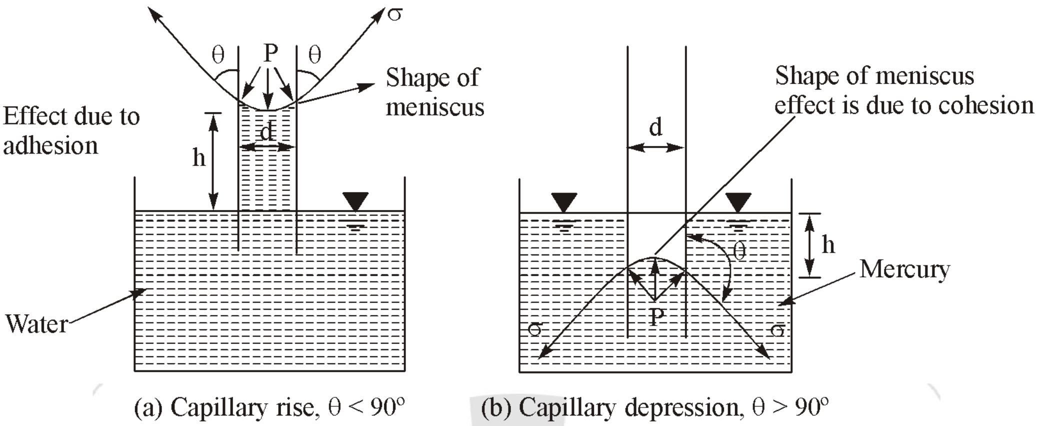
$\theta < 90^0$
Wetting liquid
Adhesion > cohesion



$\theta = 90^0$
Non-wetting liquid
Adhesion < cohesion

$$\boxed{\cos\theta = \frac{\sigma_{sa} - \sigma_{LS}}{\sigma_{aL}}}$$ Young's equation

1.8.2 Expression of Capillary Rise



Capillary rise and fall in glass tube

Let,
d = diameter of glass tube open at both ends.
h = height of the liquid in the tube
σ = surface tension of liquid
θ = angle of contact between liquid and glass tube.

Under the state of equilibrium, the weight of liquid of height h in the tube is balanced by the forces at the surface of the liquid in the tube due to surface tension.

$$\text{Weight of liquid of height h in the tube} = mg = \rho \forall g = \frac{\pi}{4} d^2 h\rho g$$

where, ρ = density of liquid.

Vertical component of the force due to surface tension = $\sigma \times \pi d \times \cos\theta$

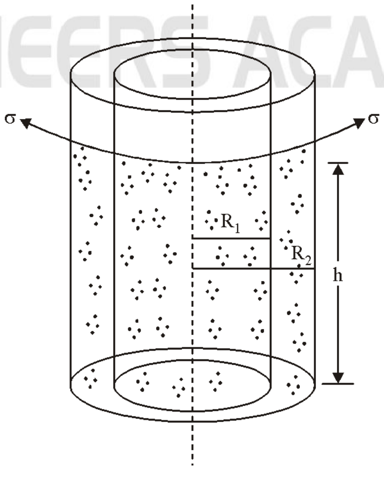
For equilibrium

$$\therefore \quad \frac{\pi}{d} d^2 h\rho g = \sigma\pi d\cos\theta$$

$$\boxed{h = \frac{4\sigma\cos\theta}{\rho g d} = \frac{4\sigma\cos\theta}{\gamma d}}$$

Note : For clean water take θ = 0

1.8.3 Annular Capillary Tube



σ same because liquid same so θ also same.

Force Balancing in Vertical Direction

$$\sigma\cos\theta \times 2\pi R_1 + \sigma\cos\theta \times 2\pi R_2 = \text{weight of the liquid}$$

$$= \rho\forall g$$

$$= \rho\left[\pi R_1^2 - \pi R_2^2\right]hg$$

$$2\sigma\cos\theta\left(R_1 + R_2\right) = \rho\pi\left(R_1 + R_2\right)\left(R_1 - R_2\right)hg$$

$$h = \frac{2\sigma\cos\theta}{\rho g\left(R_1 - R_2\right)}$$

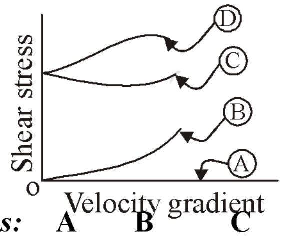
❍❍❍

PRACTICE SHEET

OBJECTIVE QUESTIONS

1. Match List-I (Curves labelled A, B, C and D in figure) with List-II (Type of fluid) and select the correct answer using the codes given below the lists:

List-I



List-II

1. Ideal plastic
2. Ideal
3. Non-Newtonian
4. Rheopectic
5. Thixotropic

Codes:

	A	B	C	D
(a)	2	3	1	5
(b)	3	2	5	1
(c)	4	2	5	1
(d)	2	3	5	1

2. If bulk modulus of water is 2×10^8 kgf/cm^2 (19.62×10^8 N/m^2), then water hammer wave celerity through a rigid pipe line will be

(a) 600 m/s (b) 800 m/s
(c) 1200 m/s (d) 1400 m/s

3. Surface tension is due to

(a) Cohesion and adhesion
(b) cohesion only
(c) Adhesion only
(d) None of the above

4. Continuum approach in fluid mechanics is valid when

(a) The compressibility is very high
(b) The viscosity is low
(c) The mean free path of the molecule is much smaller compared to the characteristic dimension
(d) $M >> 1$, where M is the Mach number

5. A fluid is said to be Newtonian fluid when the shear stress is

(a) directly proportional to the velocity gradient
(b) inversely proportional to the velocity gradient
(c) independent of the velocity gradient
(d) none of the above

6. A fluid is one which can be defined as a substance that:

(a) has same shear stress at all points
(b) can deform indefinitely under the action of the smallest shear force
(c) has the small shear stress in all directions
(d) is practically incompressible

7. The dimension of surface tension is:

(a) ML^{-1} (b) L^2V^{-1}
(c) $ML^{-1}\,T^{-1}$ (d) MT^{-2}

8. A dimensionless combination of pressure drop ΔP, dynamic viscosity μ, velocity V and length L is

(a) $\frac{\Delta P}{V^2.L}$ (b) $\frac{VL}{\mu}$
(c) $\frac{\Delta P}{\mu.VL}$ (d) $\frac{\Delta P.L}{\mu.V}$

9. Shear stress in the Newtonian fluid is proportional to

(a) pressure
(b) strain
(c) strain rate
(d) the inverse of the viscosity

11. With increase of temperature, viscosity of a fluid

(a) Does not change
(b) Always increases
(c) Always decreases
(d) Increases, if the fluid is a gas and decreases, if it is a liquid

12. The unit of dynamic viscosity of a fluid is

(a) m^2/s (b) $\frac{N \cdot s}{m^2}$
(b) $\frac{Pa \cdot s}{m^2}$ (d) $\frac{kg \cdot s^2}{m^2}$

13. The unit of surface tension is:

(a) N/m^2 (b) J/m

(c) J/m^2 (d) W/m

14. If 'P' is the gauge pressure within a spherical droplet, then gauge pressure within a bubble of the same fluid and of same size will be:

(a) $\frac{P}{4}$ (b) $\frac{P}{2}$

(c) P (d) 2P

15. Kinematic viscosity of air at 20° C is given to be $1.6 \times 10^{-5}\ m^2/s$. Its kinematic viscosity at 70° C will be varying approximately:

(a) $2.2 \times 10^{-5}\ m^2/s$ (b) $1.6 \times 10^{-5}\ m^2/s$

(c) $1.2 \times 10^{-5}\ m^2/s$ (d) $3.2 \times 10^{-5}\ m^2/s$

16. Match List-I (Fluid properties) with List-II (Related terms) and select the correct answer using the codes given below the lists:

List-I	List-II
A. Capillarity	**1.** Cavitation
B. Vapour pressure	**2.** Density of water
C. Viscosity	**3.** Shear forces
D. Specific gravity	**4.** Surface tension

Codes:	A	B	C	D
(a)	1	4	2	3
(b)	1	4	3	2
(c)	4	1	2	3
(d)	4	1	3	2

17. Which one of the following is the bulk modulus K of a fluid? (Symbols have the usual meaning)

(a) $\rho\frac{dp}{d\rho}$ (b) $\frac{dp}{\rho d\rho}$

(c) $\rho\frac{d\rho}{dp}$ (d) $\frac{dp}{\rho dp}$

18. Which of the following forces act on a fluid at rest?

1. Gravity force

2. Hydrostatic force

3. Surface tension

4. Viscous force

Select the correct answer using the codes given below:

(a) 1, 2, 3 and 4 (b) 1, 2 and 3

(c) 2 and 4 (d) 1, 3 and 4

19. Surface tension is due to

(a) viscous forces

(b) cohesion

(c) adhesion

(d) the difference between adhesive and cohesive forces

20. Newton's law of viscosity depends upon the

(a) stress and strain in a fluid

(b) shear stress, pressure and velocity

(c) shear stress and rate of strain

(d) viscosity and shear stress

21. If the surface tension of water-air interface is 0.073 N/m, the gauge pressure inside a rain drop of 1 mm diameter will be

(a) $0.146\ N/m^2$

(b) $73\ N/m^2$

(c) $146\ N/m^2$

(d) $292\ N/m^2$

24. If the volume of a liquid decreases by 0.2 percent for an increase of pressure from $6.867\ MN/m^2$ to $15.696\ MN/m^2$, what is the value of the bulk modulus of the liquid ?

(a) 4418 MPa (b) 441.8 MPa

(c) 4.418 MPa (d) 44180. MPa

25. If a certain liquid has a viscosity of 0.048 poise and kinematic viscosity 3.50×10^{-2} stokes, what is its specific gravity?

(a) 1.23142 (b) 1.001

(c) 1.37142 (d) 1.17353

* * *

ANSWERS AND EXPLANATIONS

1. ***Ans. (a)***

Horizontal line representing zero shear stress for any velocity gradient is the condition for ideal fluid.

The curve B represents dilatant fluid.

$$\boxed{\tau = \mu\left(\frac{du}{dy}\right)^n \; n > 1}$$

For pseudoplastic fluid, $\boxed{\tau = \mu\left(\frac{du}{dy}\right)^n \; n < 1}$

For thixotropic fluid, $\boxed{\tau = \tau_0 + \mu\left(\frac{du}{dy}\right)^n \; n < 1}$

For rheopectic fluid, $\boxed{\tau = \tau_0 + \mu\left(\frac{du}{dy}\right)^n \; n > 1}$

For plastic fluid, $\boxed{\tau = \tau_0 + \mu\left(\frac{du}{dy}\right) \text{i.e., } n = 1}$

2. ***Ans. (d)***

$$C = \sqrt{\frac{k}{\rho}} = \sqrt{\frac{19.62 \text{ x } 10^8}{1000}} = 1400 \text{ m/s}$$

K = bulk modulus of elasticity

ρ = mass density of water

3. ***Ans. (b)***

Surface tension is due to cohesive force only.

4. ***Ans. (c)***

Continuum approach means properties uniform through out the medium. This condition is valid only when mean free path (λ) is much lower as compare to characteristic length (L_C).

5. ***Ans. (a)***

For newtonian fluid

$$\boxed{\tau \alpha \frac{du}{dy} \alpha \frac{d\theta}{dt}}$$

6. ***Ans. (b)***

Fluid is substance which deform continuously under the action of small shear or tangential force.

7. ***Ans. (d)***

Surface tension = $\frac{F}{\ell}$

$$\boxed{= \left(\frac{N}{m}\right) = \frac{kg \cdot m / s^2}{m} = [M^1L^0T^{-2}]}$$

8. ***Ans. (d)***

$$\Delta P = N/m^2 = kg.\frac{m}{sec^2}/m^2 = M^1.L^{-1}.T^{-2}$$

$$V = m/s = L^1T^{-1}$$

$$L = M = L^1$$

$$\mu = \frac{N-S}{m^2} = kg/m-s = M^1.L^{-1}T^{-1}$$

$$\mu = \frac{N-S}{m^2} = kg/m-s = m^1.L^{-1}T^{-1}$$

Check:

(a) $\frac{\Delta P}{V^2 \cdot L} = \frac{M^1L^{-1}T^{-2}}{L^2T^{-2}.L^1} = M^1L^{-4}$

(b) $\frac{V.L}{\mu} = \frac{L^1T^{-1}L^1}{M^1L^{-1}T^{-1}} = M^{-1}L^{-4}$

(c) $\frac{\Delta P}{\mu.V.L.} = \frac{M^1L^{-1}T^{-2}}{M^1L^{-1}T^{-1}.L^1T^{-1}L^1} = L^{-2}$

(d) $\frac{\Delta P.L}{\mu.V} = \frac{M^1L^{-1}T^{-2}.L^1}{M^1L^{-1}T^{-1}.L^1T^{-1}} = M^0L^0T^0$

9. ***Ans. (c)***

$$\tau \alpha \frac{du}{dy} \alpha \frac{d\theta}{dt}$$

where, θ is angular strain

$\frac{du}{dy}$ has it's unit is s^{-1} i.e., rate of strain.

11. ***Ans. (d)***

Viscosity is due to cohesion in liquids. As a temperature increases cohesion of liquids decrease. Hence as $T(\uparrow)\mu$ of liquids decrease. In case of gases, viscosity depends on molecular momentum exchange. As temperature, increases, molecular activity of gases increase and hence resistance to flow increase. Hence as temperature increases viscosity of gases increases.

12. ***Ans. (b)***

$$\tau = \mu\frac{du}{dy}$$

$$\frac{N}{m^2} = \mu\frac{m/s}{m}$$

$$\boxed{\mu = \left(\frac{N-s}{m^2}\right)}$$

13. ***Ans. (c)***

Surface tension $= \frac{F}{\ell} = \frac{N}{m}\times\frac{m}{m} = J/m^2$

14. ***Ans. (d)***

$\Delta P = \frac{4\sigma}{D}$ in case of drop.

$\Delta P = \frac{8\sigma}{D}$ in case of bubble

and hence $(\Delta P)_{bubble} = 2\ (\Delta P)_{drop}$

15. ***Ans. (a)***

For Air

$$\mu \propto \sqrt{T}\ \ \&\ \ \rho \propto 1/T$$

$$v = \frac{\mu}{\rho}$$

$$v \propto \frac{\sqrt{T}}{1/T}$$

$$v \propto T^{3/2}$$

16. ***Ans. (d)***

Viscosity : It is a measure of resistance of a fluid which is being deformed by either shear stress or tensile stress.

Specific gravity : It is the ratio of density of fluid to the density of standard fluid.

Capillarity : It is the ability of liquid to flow against gravity combination of surface tension and adhesion act to lift the liquid.

17. ***Ans. (a)***

Bulk modulus $K = -\frac{dp}{d\forall/\forall}$

and $\forall = \frac{m}{\rho}$

$\therefore \quad d\forall = -\frac{d\rho}{\rho^2}$

$$K = -\frac{dp}{\frac{-d\rho/\rho^2}{1/\rho}}$$

$$K = \frac{\rho dp}{d\rho}$$

18. ***Ans. (b)***

A fluid at rest there can be no shear force (i.e. viscous force). The only forces acting on the free body are the normal pressure forces, exerted by the surrounding fluid on the plane surface and the weight of the element.

19. ***Ans. (b)***

Surface tension is due to cohesion between liquid particles at the surface, where as capillarity is due to both cohesion and adhesion.

The property of cohesion enables a liquid to resist tensile stress, while adhesion enables it to stick to another body.

20. ***Ans. (c)***

Newton's law of viscosity

$\tau = \mu\frac{du}{dy}$ Where τ = shear stress

$\frac{du}{dy}$ = Rate of strain

21. ***Ans. (d)***

Pressure intensity inside a droplet

$$\Delta P = \frac{4\sigma}{d} = \frac{4\times 0.073}{10^{-3}} N/m^2 = 292\,N/m^2$$

24. ***Ans. (a)***

$$\frac{\Delta V}{V} = -0.002$$

$$P_1 = 6.867\times 10^6\,\frac{N}{m^2}$$

$$P_2 = 15.696\times 10^6\,\frac{N}{m^2}$$

hence bulk modulus is

$$K = \frac{-dp}{d\forall \mid \forall} = \frac{\left(15.696 \times 10^6 - 6.86 \times 10^6\right)}{0.002}\frac{N}{m^2}$$

$$= 4.418 \times 10^9 \frac{N}{m^2}$$

$$= 4418\,MN / m^2$$

25. ***Ans. (c)***

Given that

$$\mu(\text{dynamic cis cosity}) = 0.048 \times 10^{-1} \frac{N-s}{m^2}$$

$$\upsilon = 3.50 \times 10^{-2} \text{ stroke}$$

$$= 3.50 \times 10^{-6} \text{ m}^2/\text{sec}$$

we know that

$$\frac{\mu}{\rho} = \upsilon$$

$$\upsilon = \left(\frac{\mu}{\rho}\right) = \frac{0.048 \times 10^{-1} \frac{N-s}{m^2}}{3.50 \times 10^{-6} m^2/\text{sec}}$$

$$\rho = 1.31742 \text{ kg} / m^3$$

$$S = \frac{\rho}{1000}$$

$$S = 1.31742$$

* * *