CONTENTS

S.No.	Topic	Page No.
1.	Introduction	1 – 1
2.	System of Forces	2 – 9
3.	Friction & Application	10 – 17
4.	Center of Gravity	18 – 25
5.	Moment of Inertia	26 – 32
6.	Kinematics	33 – 43
7.	Trusses	44 – 54
8.	Collisions	55 – 62
9.	Rigid Body Dynamics in Plane Motion	63 – 70
10.	Objective Sheet	71 – 87

3

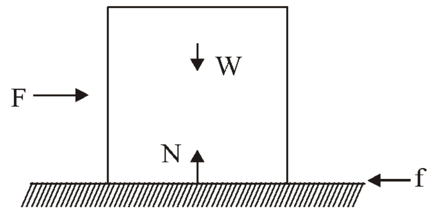
FRICTION & APPLICATION

THEORY

3.1 FRICTION

It is the resistance force between contact surfaces due to roughness of surfaces

It always acts opposite to the relative motion of contact surfaces



F → applied force

W → weight of body

N → Normal Reaction

f → Friction or frictional force.

3.2 LAW OF FRICTION

Maximum frictional force is directly proportional to the normal force between the surfaces

$$f = \mu N$$

μ → coefficient of friction.

$$(f_s)_{max} = \mu_s N$$

$(f_s)_{max}$→ Limiting friction.

When the applied force is less than Maximum friction force . The body is in rest and the friction forces is equal to applied force.

When applied force is equal to maximum friction force, the body is about to move, this condition is called impending motion.

When applied force is more than maximum friction force, the body is in motion and friction on the body is kinetic friction.

$$(f_k) = \mu_k N$$

3.3 Application of Friction

(1) Wedges

(2) Ladder

(3) Screw Jack

(4) Brakes & Clutches

Example 1

The coefficient of static friction between a block of mass m and an incline is $\mu_s = 0.3$. (a) What can be the maximum angle θ of the incline with the horizontal so that the block does not slip on the plane? (b) If the incline makes an angle θ/2 with the horizontal, find the frictional force on the block.

Solution:

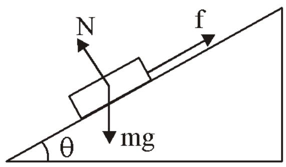
The situation is shown in figure.

(a) The forces on the block are

(i) the weight mg downward by the earth,

(ii) the normal contact force N by the incline, and

(iii) the friction 'f' parallel the incline up the plane, by the incline.



As the block is at rest, these forces should add up to zero. Also, since θ is the maximum angle to prevent slipping, this is a case of limiting equilibrium and so $f = \mu_s N$.

Taking components perpendicular to the incline,

$$N - mg\cos\theta = 0$$

or, $$N = mg\cos\theta \quad ....(i)$$

Taking components parallel to the incline,

$$f - mg\sin\theta = 0$$

or, $$f = mg\sin\theta$$

or, $$\mu_s N = mg\sin\theta \quad ...(ii)$$

Dividing (ii) by (i) $\mu_s = \tan\theta$

or, $$\theta = \tan^{-1}\mu_s = \tan^{-1}(0.3).$$

(b) If the angle of incline is reduced to θ/2, the equilibrium is not limiting, and hence the force of static friction f is less than $\mu_s N$. To know the value of f, we proceed as in part (a) and get the equations

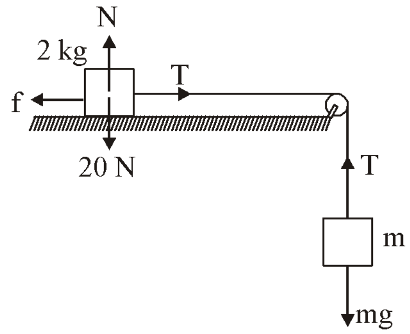
$$N = mg\cos(\theta/2)$$

and $$f = mg\sin(\theta/2).$$

Thus, the force of friction is mg sin (θ/2).

Example 2

The coefficient of static friction between the block of 2 kg and the table shown in figure is $\mu_s = 0.2$. What should be the maximum value of m so that the blocks do not move ? Take $g = 10 \text{ m/s}^2$. The string and the pulley are light and smooth.



Solution:

Consider the equilibrium of the block of mass m. The forces on this block are

(a) mg downward by the earth and

(b) T upward by the string.

Hence, $$T - mg = 0 \text{ or, } T = mg. \quad ...(i)$$

Now consider the equilibrium of the 2 kg block. The forces on this block are

(a) T towards right by the string.

(b) f towards left (friction) by the table,

(c) 20 N downward (weight) by the earth and

(d) N upward (normal force) by the table.

For vertical equilibrium of this block,

$$N = 20 \text{ N}. \quad ...(ii)$$

As m is the largest mass which can be used without moving the system, the friction is limiting.

Thus, $$f = \mu_s N. \quad ...(iii)$$

For horizontal equilibrium of the 2 kg block,

$$f = T. \quad ...(iv)$$

Using equations (i), (iii), and (iv)

$$\mu_s N = mg$$

or, $$0.2 \times 20 \text{ N} = mg$$

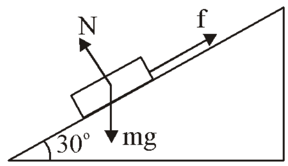
or, $$m = \frac{0.2 \times 20}{10} \text{ kg} = 0.4 \text{ kg}.$$

Example 3

A block slides down an incline of angle 30° with an acceleration g/4. Find the kinetic friction coefficient.

Solution:

Let the mass of the block be m. The forces on the block are



(a) mg downward by the earth (gravity),

(b) N normal force by the incline and

(c) f up the plane, (friction) by the incline.

Taking components parallel to the incline and writing Newton's second law,

$$mg \sin 30^{o} - f = mg/4$$

$$f = mg/4.$$

There is no acceleration perpendicular to the incline.

Hence

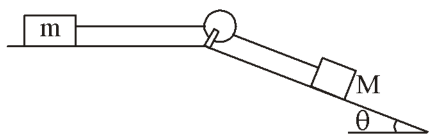
$$N = mg \cos 30^{o} = mg. \frac{\sqrt{3}}{2}$$

As the block is slipping on the incline, friction is $f = \mu_k N$.

So $$\mu_k = \frac{f}{N} = \frac{mg}{4mg\sqrt{3}/2} = \frac{1}{2\sqrt{3}}.$$

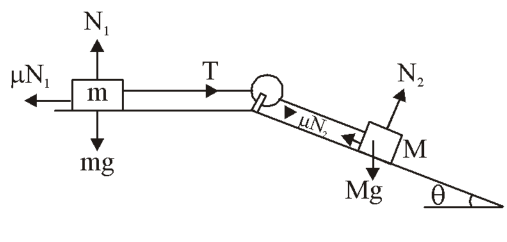
Example 4

Find the maximum value of M/m in the situation shown in figure so that the system remains at rest. Friction coefficient at both the contacts is μ. Discuss the situation when $\tan\theta < \mu$.



Solution:

Figure shows the forces acting on the two blocks. As we are looking for the maximum value of M/m,. the equilibrium is limiting. Hence, the frictional forces are equal to μ times the corresponding normal forces.



Equilibrium of the block m gives

$$T = \mu N_1 \text{ and } N_1 = mg$$

which gives

$$T = \mu \text{ mg.} \quad ...(i)$$

Next, consider the equilibrium of the block M. Taking components parallel to the incline.

$$T + \mu N_2 = Mg \sin\theta.$$

Taking components normal to the incline

$$N_2 = Mg \cos\theta.$$

These give $\quad T = Mg(\sin\theta - \mu \cos\theta). \quad ...(ii)$

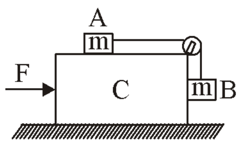
From (i) and (ii), $\quad \mu mg = Mg (\sin\theta - \mu \cos\theta)$

or, $\quad M/m = \dfrac{\mu}{\sin\theta - \mu\cos\theta}$

If $\tan\theta < \mu$, $(\sin\theta - \mu \cos\theta) < 0$ and the system will not slide for any value of M/m.

Example 5

Consider the situation shown in figure . The horizontal surface below the bigger block is smooth. the coefficient of friction between the blocks is μ. Find the minimum and the maximum force F that can be applied in order to keep the smaller blocks at rest with respect to the bigger block.

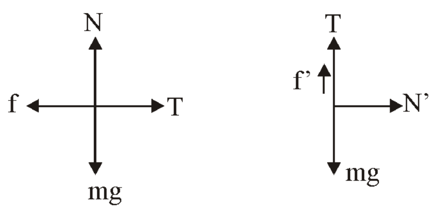


Solution:

If no force is applied, the block A will slip on C towards right and the block B will move downward. Suppose the minimum force needed to prevent slipping if F. Taking A + B+ C as the system, the only external horizontal force on the system is F. Hence, the acceleration of the system is

$$a = \frac{F}{M+2m} \quad ...(i)$$

Now take the block A as the system. The forces on A are,



(i) tension T by the string towards right,

(ii) friction f by the block C towards left,

(iii) weight mg downward and

(iv) normal force N upward.

For vertical equilibrium N = mg

As the minimum force needed to prevent slipping is applied, the friction is limiting. Thus,

$$f = \mu N = \mu\, mg.$$

As the block moves towards right with an acceleration a,

$$T - f = ma$$

or, $$T - \mu\, mg = ma \qquad \text{...(ii)}$$

Now take the block B as the system. The forces are,

(i) tension T upward,

(ii) weight mg downward,

(iii) normal force N towards right, and

(iv) friction f' upward.

As the block moves towards right with an acceleration a,

$$N' = ma.$$

As the friction is limiting, $f' = \mu N' = \mu ma$.

For vertical equilibrium

$$T + f' = mg$$

or, $$T + \mu\, ma = mg. \qquad \text{...(iii)}$$

Eliminating T from (ii) and (iii)

$$a_{min} = \frac{1-\mu}{1+\mu} g.$$

When a large force is applied the block A slips on C towards left and the block B slips on C in the upward direction. The friction on A is towards right and that on B is downwards. Solving as above, the acceleration in this case is

$$a_{max} = \frac{1+\mu}{1-\mu} g.$$

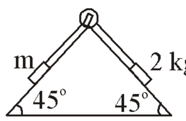
Thus, a lies between $\frac{1-\mu}{1+\mu} g$ and $\frac{1+\mu}{1-\mu} g$.

From (i) the force F should be between

$\frac{1-\mu}{1+\mu}(M + 2m)\, g$ and $\frac{1+\mu}{1-\mu}(M + 2m)\, g$.

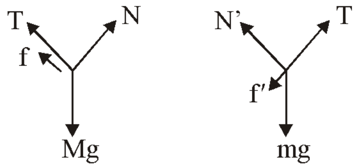
Example 6

Figure shows two blocks connected by a light string placed on the two inclined parts of a triangular structure. The coefficients of static and kinetic friction are 0.28 and 0.25 respectively at each of the surfaces. (a) Find the minimum and maximum values of m for which the system remains at rest. (b) Find the acceleration of either block if m is given the minimum value calculated in the first part and is gently pushed up the incline for a short while



Solution:

(a) Take the 2 kg block as the system. The forces on this block are shown in figure with M = 2 kg. It is assumed that m has its minimum value so that the 2 kg block has a tendency to slip down. As the block is in equilibrium, the resultant force should be zero.



Taking components ⊥ to the incline

$$N = Mg \cos 45^o = Mg/\sqrt{2}.$$

Taking components || to the incline

$$T + f = Mg \sin 45^o = Mg/\sqrt{2}$$

or, $$T = Mg/\sqrt{2} - f.$$

As it is a case of limiting equilibrium,

$$f = \mu_s N$$

or, $$T = \frac{Mg}{\sqrt{2}} - \mu_s \frac{Mg}{\sqrt{2}} = \frac{Mg}{\sqrt{2}}(1 - \mu_s). \quad \text{...(i)}$$

Now consider the other block as the system. The forces acting on this block are shown in figure. Taking components ⊥ to the incline,

$$N' = mg \cos 45^o = mg/\sqrt{2}.$$

Taking components || to the incline

$$T = mg \sin 45^o + f' = \frac{mg}{\sqrt{2}} + f'.$$

As it is the case of limiting equilibrium

$$f' = \mu_s N' = \mu_s \frac{mg}{\sqrt{2}}.$$

Thus, $$T = \frac{mg}{\sqrt{2}}(1 + \mu_s), \quad \text{...(ii)}$$

From (i) and (ii)

$$m(1 + \mu_s) = M(1 - \mu_s) \quad \text{...(iii)}$$

or, $$m = \frac{(1-\mu_s)}{(1+\mu_s)} M = \frac{1-0.28}{1+0.28} \times 2 \text{ kg}$$

$$= \frac{9}{8} \text{ kg}.$$

When maximum possible value of m is supplied, the directions of friction are reversed because m has the tendency to slip down and 2 kg block to slip up. Thus, the maximum value of m can be obtained from (iii) by putting $\mu_s = -0.28$. Thus, the maximum value of m is

$$m = \frac{1+0.28}{1-0.28} \times 2 \text{ kg}$$

$$= \frac{32}{9} \text{kg.}$$

(b) If m = 9/8 kg and the system is gently pushed, kinetic friction will operate. Thus,

$$f = \mu_k \cdot \frac{Mg}{\sqrt{2}} \text{ and } f' = \frac{\mu_k mg}{\sqrt{2}},$$

where $\mu_k = 0.25$. If the acceleration is a, Newton's second law for M gives.

$$Mg \sin 45^\circ - T - f = Ma$$

or, $$\frac{Mg}{\sqrt{2}} - T - \frac{\mu_k Mg}{\sqrt{2}} = Ma. \quad \text{...(iv)}$$

Applying Newton's second law

$$T - mg \sin 45^\circ - f' = ma$$

or, $$T - \frac{mg}{\sqrt{2}} - \frac{\mu_k mg}{\sqrt{2}} = ma. \quad \text{...(v)}$$

Adding (iv) and (v)

$$\frac{Mg}{\sqrt{2}}(1 - \mu_k) - \frac{mg}{\sqrt{2}}(1 + \mu_k) = (M + m)\, a$$

or, $$a = \frac{M(1-\mu_k) - m(1-\mu_k)}{\sqrt{2}(M+m)} g$$

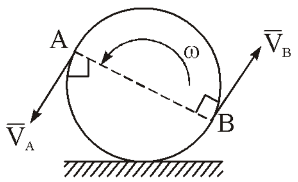
$$= \frac{2 \times 0.75 - 9/8 \times 1.25}{\sqrt{2}(2+9/8)} g$$

$$= 0.31 \text{ m/s}^2$$

❑❑❑

OBJECTIVE SHEET

1. A and B are the end-points of a diameter of a disc rolling along a straight line with a counter Clock-wise angular velocity as shown in the figure. Referring to the velocity vectors $\overline{V}_A$ and $\overline{V}_B$ shown in the figure:



(a) $\overline{V}_A$ and $\overline{V}_B$ are both correct.

(b) $\overline{V}_A$ is incorrect but $\overline{V}_B$ is correct.

(c) $\overline{V}_A$ and $\overline{V}_B$ are both incorrect.

(d) $\overline{V}_A$ is correct but $\overline{V}_B$ is incorrect

2. Instantaneous centre of a body rolling with sliding on a stationary curved surface lies.

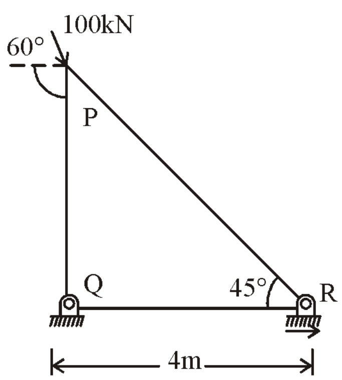
(a) at the point of contact

(b) on the common normal at the point of contact

(c) on the common tangent at the point of contact

(d) at the centre of curvature of the stationary surface.

3. For the truss shown in figure, the magnitude of the force in member *PR* and the support reaction at *R* are respectively



(a) 122.47 kN and 50 kN

(b) 70.71 kN and 100 kN

(c) 70.71 kN and 50 kN

(d) 81.65 kN and 100 kN

4. A stone of mass m at the end of a string of length L is whirled in a vertical circle at a constant speed. The tension in the string will be maximum when the stone is

(a) at the top of the circle

(b) half way down from the top

(c) quarter-way down from the top

(d) at the bottom of the circle

5. A ball A of mass m falls under gravity from a height h and strikes another ball B of mass m which is supported at rest on a spring of stiffness k. Assume perfect elastic impact. Immediately after the impact

(a) the velocity of ball A is $\frac{1}{2}\sqrt{2gh}$

(b) the velocity of ball A is zero

(c) the velocity of both balls is $\frac{1}{2}\sqrt{2gh}$

(d) none of the above

6. A wheel of mass m and radius R is in accelerated rolling motion without slip under a steady axle torque T. If the coefficient of kinetic friction is μ, the friction force from the ground on the wheel is

(a) μmg (b) T/R

(c) zero (d) none of the above

7. A car moving with uniform acceleration covers 450m in a 5 second interval, and covers 700m in the next 5 second interval. The acceleration of the car is

(a) $7m/s^2$ (b) $50m/s^2$

(c) $25m/s^2$ (d) $10m/s^2$

8. A steel wheel of 600 mm diameter on a horizontal steel rail. It carries a load of 500 N. The coefficient of rolling resistance is 0.3. The force in Newton, necessary to roll the wheel along the rail is

(a) 0.5 (b) 5

(c) 1.5 (d) 150

9. The ratio of tension on the tight side to that on the slack ride in a flat belt drive is

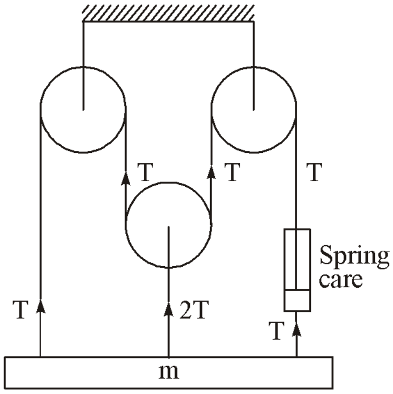
(a) proportional to the product of coefficient of friction and lap angle.

(b) an exponential function of the product of coefficient of friction and lap angle.

(c) proportional to lap angle

(d) proportional to the coefficient of friction.

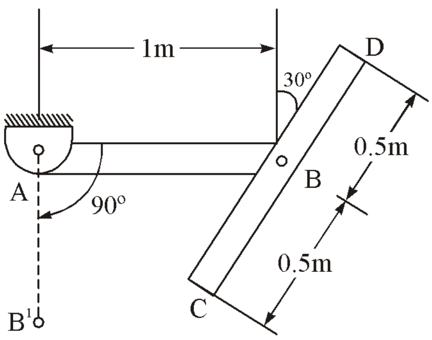
10. A spring scale indicates a tension T in the right hand cable of the pulley system shown in Neglecting the mass of the pulleys and ignoring friction between the cable and pulley the mass m is



(a) 2T/g (b) T $(1+C^{4\pi})$/g

(c) 4T/g (d) none of the above

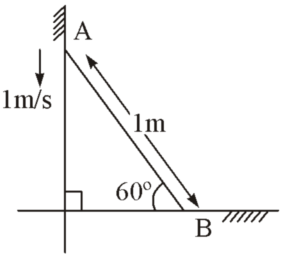
11. AB and CD are two uniform and identical bars of mass 10kg each, as shown. The hinges at A and B are frictionless. The assembly is released from rest and motion occurs in the vertical plane. At the instant that the hinge B passes the point B, the angle between the two bars will be



(a) 60 degrees (b) 37.4 degrees

(c) 30 degrees (d) 45 degrees

12. A rod of length 1m is sliding in a corner as shown in. At an instant when the rod makes an angle of 60 degrees with the horizontal plane, the velocity of point A on the rod is 1m/s. The angular velocity of the rod at this instant is



(a) 2 rad/s (b) 1.5 rad/s

(c) 0.5 rad/s (d) 0.75 rad/s

13. Match 4 correct pairs between List-I and List-II.

List-I

(a) Collision of particles

(b) Stability

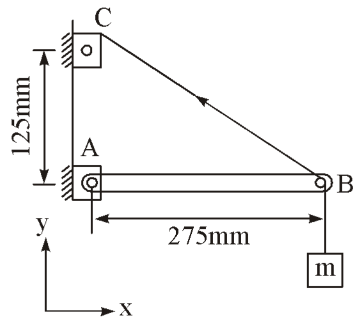
(c) Satellite motion

(d) Spinning top

List-II

1. Euler's equation of motion
2. Minimum kinetic energy
3. Minimum potential energy
4. Impulse-momentum principle
5. Conservation of moment of momentum

14. A mass 35 kg is suspended from a weightless bar AB which is supported by a cable CB and a pin at A as shown in. The pin reactions at A on the bar AB are



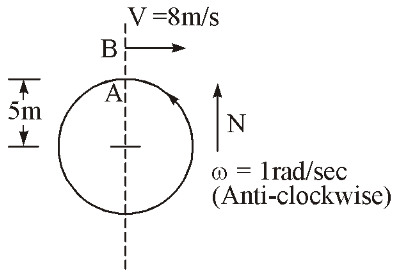
(a) R_x = 343.4N, R_y = 755.4N

(b) R_x = 343.4N, R_y = 0

(c) R_x = 755.4N, R_y = 343.4N

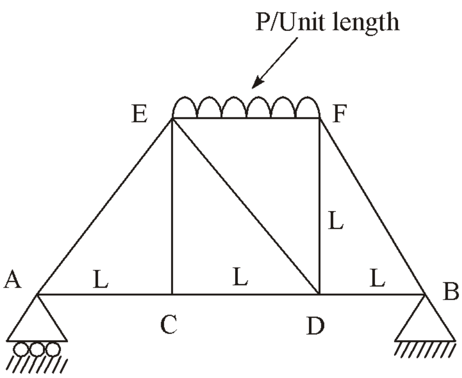
(d) R_x = 755.4N, R_y = 0

15. As shown in Figure, a person A is standing at the top of a rotating platform facing person B who is riding a bicycle, heading East. The relevant speeds and distances are shown in given figure person, a bicycle, heading East. At the instant under consideration, what is the apparent velocity of B as seen by A?



(a) 3 m/s heading East
(b) 3 m/s heading West
(c) 8 m/s heading East
(d) 13 m/s heading East

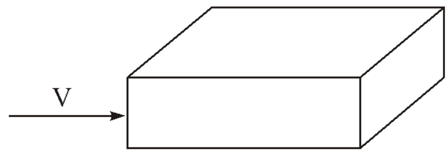
16. A truss consists of horizontal members and vertical members having length L each. The members AE, DE and BF are inclined at 45° to the horizontal. For the uniformly distributed load P per unit length on the member EF of the truss shown in the figure, the force in the member CD is



(a) $\frac{PL}{2}$ (b) PL

(c) Zero (d) $\frac{2PL}{3}$

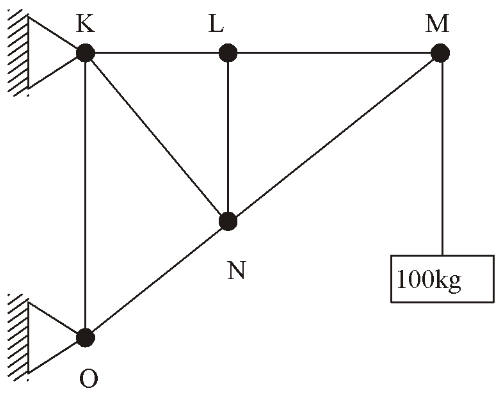
17. A bullet of mass 'm' travels at a very high velocity 'V' (as shown in the figure) and gets embedded inside the block of mass 'M' initially at rest on a rough horizontal floor. The block with the bullet is seen to move a distance 'S' along the floor. Assuming μ to be the coefficient of kinetic friction between the block and the floor and 'g' the acceleration due to gravity, what is the velocity 'V' of the bullet?



(a) $\frac{M+m}{m}\sqrt{2\mu gs}$ (b) $\frac{M-m}{m}\sqrt{2\mu gs}$

(c) $\frac{\mu(M+m)}{m}\sqrt{2\mu gs}$ (d) $\frac{M}{m}\sqrt{2\mu gs}$

18. The figure shows a pin-jointed plane truss loaded at the point M by hanging a mass of 100 kg. The member LN of the truss is subjected to a load of



(a) Zero
(b) 490 N in compression
(c) 981 N in compression
(d) 98I N in tension

19. Two books of mass 1 kg each are kept on a table one over the other. The coefficient of friction on every pair of containing surfaces is 0.3. The lower book is pulled with a horizontal force F. The minimum value of F for which slip occurs between the two books and ground is?

(a) Zero (b) 1.06 N
(c) 5.74 N (d) 8.83 N