THEORY & OBJECTIVE

THEORY OF STRUCTURES

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DETERMINACY INDETERMINACY

THEORY

1.1 Equation of Static Equilibrium

In a 2-D structure or plane structure (in which all members and forces are in one plane only), the equation of equilibrium are

$$\left.\begin{array}{r}\Sigma F_x = 0\\ \Sigma F_y = 0\\ \Sigma M_z = 0\end{array}\right\} \text{3 number}$$

In a 3-D structure or space structure (in which members and forces are in 3-D), the equations of equilibrium are

$$\left.\begin{array}{ll}\Sigma F_x = 0 & \Sigma M_x = 0\\ \Sigma F_y = 0 & \Sigma M_y = 0\\ \Sigma F_z = 0 & \Sigma M_z = 0\end{array}\right\} \text{6 number}$$

If member forces cannot be found by equations of static equilibrium alone, the structure is called statically indeterminate.

In this case additional equation needed are obtained by relating the applied loads and reactions to the displacement or slopes known at different points on the structure. These equations are called **compatibility equations.**

1.2 Degree of Static Indeterminacy (D_S)

$$D_S = \begin{pmatrix}\text{No. of unknown forces in members}\\ \text{+ unknown support reactions}\end{pmatrix} - \begin{pmatrix}\text{Available equations of}\\ \text{static equilibrium}\end{pmatrix}$$

and $D_S = D_{Si} + D_{Se}$

Where, D_S = Total indeterminacy

D_{Si} = Degree of internal static indeterminacy

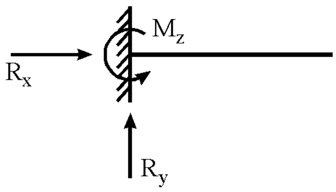
D_{Se} = Degree of external static indeterminacy

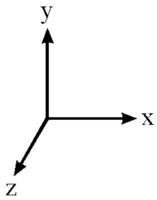
1.3 Support Reactions

Restraining of deformation at support gives rise to support reactions.

1.3.1 Plane Structure

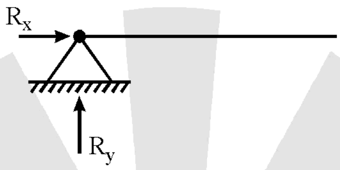
(1) Fixed support





Fixed support restrains Δ_x, Δ_y and θ_{xy}. Hence support reactions are R_x, R_y and M_z (3 nos.)

(2) Pin support or hinged support

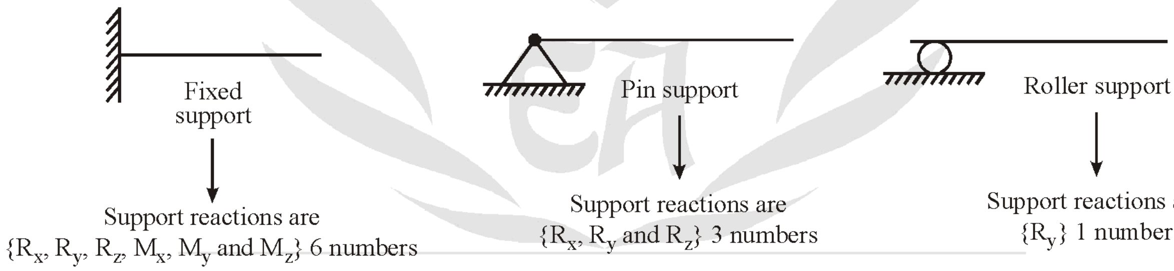


Pin support restrains Δ_x and Δ_y. Hence support reactions are R_x and R_y (2 nos.)

(3) Roller support

Roller support restrains Δ_y. Hence support reaction is R_y.

1.3.2 Space Structure



1.4 External Indeterminacy (D_{Se})

$$D_{Se} = \begin{pmatrix}\text{Total no. of support reactions} \\ \text{in the structure}\end{pmatrix} - \begin{pmatrix}\text{Available equations of} \\ \text{static equilibrium}\end{pmatrix}$$

For plane structure $D_{Se} = R_e - 3$

For space structure $D_{Se} = R_e - 6$

1.5 Internal Indeterminacy (D_{Si})

$$D_{Si} = D_S - D_{Se}$$

$$= \text{Total indeterminacy} - \text{External indeterminacy}$$

1.6 Degree of Static Indeterminacy for Frames

Frames are rigid jointed structures. All the joints are made rigid by providing extra restraint R′. The structure is then cut to make it, *Open Tree* like determinate structure.

For plane frames $D_S = 3C - R'$

For space frames $D_S = 6C - R'$

Where, C = Number of cuts to make structure determinate

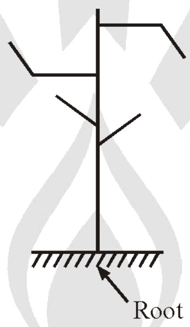
R′ = Number of restraints applied to make all joints rigid.

1.6.1 Open Tree Like Structure

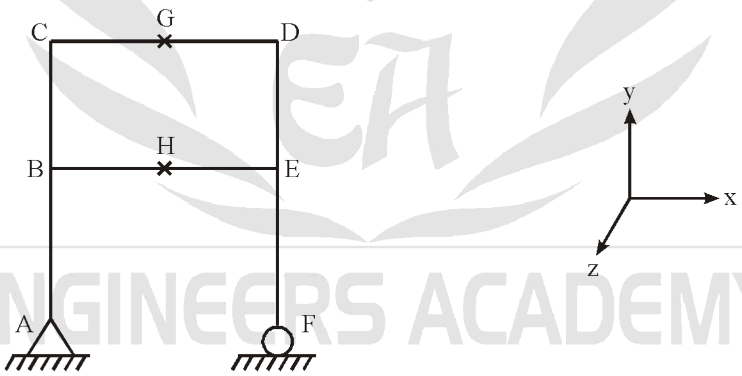
The structure is cut in such a way that each individual cut part looks like a tree as shown below.

Note that :

1. Tree should have only one root.
2. Tree cannot have a closed looped branch.



Ex.:



Since it is a plane frame.

$$D_S = 3 \times 2 - (3) = 3$$

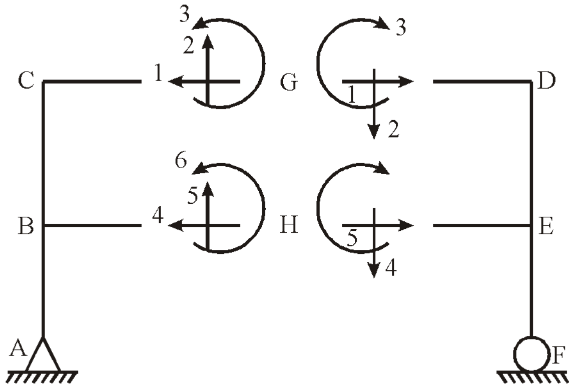
Number of restraint that is required to be applied to make the structure rigid is

1. One number at support A i.e. moment M_{Az}.
2. Two numbers at support F i.e. R_{Fx} and M_{Fz}.

Thus, total number of added restraints = $R' = 1 + 2 = 3$

This $R' = 3$ corresponds to 3 known reaction conditions i.e. $M_{Az} = 0$, $M_{Fz} = 0$ and $R_{Fx} = 0$.

When the structure is cut it gets divided into two parts.



Open tree like structure

If these 6 reactions at the cut section are known, the structure become completely determinate i.e. forces in all members AB, BC, DE, EF, CD and BE can be determined.

Thus, Number of unknowns = six reactions at the cut section – three known conditions

Where known conditions are $\Sigma R_{Fx} = 0$

$\Sigma M_{Az} = 0$

$\Sigma M_{Fz} = 0$

As the number of cuts are C = 2

Number of unknowns $= 3 \times C - R'$

$= 3 \times 2 - 3 = 3$

In 3-D frame, at any cut section number of reactions = 6 [R_x, R_y, R_z, M_x, M_y and Mz]

Hence, Degree of static indeterminacy for 3D frame = 6C – R′

1.6.2 Restraining Support

For plane and space frames

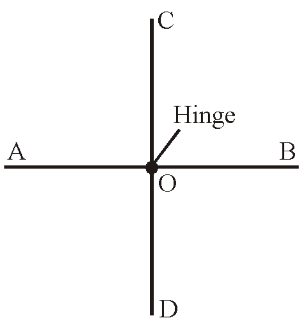
$$\begin{array}{c}\text{Number of restraint required}\\ \text{to make a joint rigid}\end{array} = \begin{pmatrix}\text{Number of support reactions}\\ \text{for fixed support}\end{pmatrix} - \begin{pmatrix}\text{Number of support reactions}\\ \text{of actual support}\end{pmatrix}$$

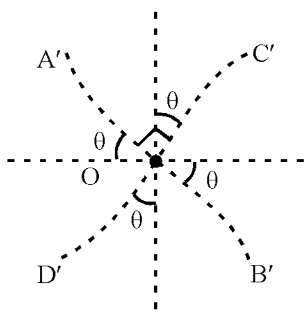
1.6.3 Restraining Member/Joint

(1) Plane Frame (Joint having hinge) : Number of restraining moments required at a joint where m-members meet = (m – 1) or no of additional equations gained.

(2) Space Frame (Joints having hinge) : Number of restraining moments required at a joint where m-members meet = 3(m – 1) or no of additional equations gained.

(3) Explanation for Applying (m – 1) Restraining Moment at Joint Having Hinge in 2D Frame : If the joint O had been rigid, rotation of one member with respect to other will be zero as shown in figure below.





However, with joint having hinge, OC, OB and OD will have rotation with respect to OA. To make these three relative rotations zero, we need to apply 3-moments. Thus for 4-members meeting at a joint, number of restraining moments required = 3 = (4 – 1).

Hence, for m-members meeting at a joint number of restraining moments required = (m – 1).

On similar line, it can be shown that in space frame, each member has 3-rotations possible in 3-different planes. Hence with respect to one member number of rotations possible are

$$3m - 3 = 3(m - 1)$$

To restrain these we need to apply 3(m – 1) moments. Hence number of restraining moments required at joint with hinge in 3D-frame = 3(m – 1).

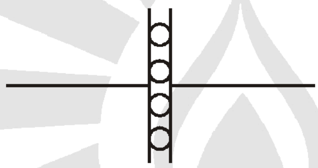
Ex. 1 :

No. of restraint required to be added = 2. They are R_H and M. R_H restrains relative movement.

Ex. 2 :

Number of restraint required to be added = 1 (R_H)

Ex. 3 :



Number of restraint required to be added = 1 (R_V)

1.6.4 Second Method (For Rigid Frame)

In plane frame, every member carries three forces i.e. BM, SF, Axial Force.

Hence Total number of unknowns = $3m + R_e$

Where, m = number of members

and R_e = number of support reactions

At each joint, number of equations of equilibrium available = 3

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M_z = 0$$

Total number of equations of equilibrium = 3j

Where, j = number of joints

Hence, degree of static indeterminacy

$$D_S = R_e + 3m - 3j$$

However if the frame carries hinges, then D_S is reduced further by $\Sigma(m' - 1)$, where m′ = number of members meeting at the hinge.

$$D_S = R_e + 3m - 3j - \Sigma(m' - 1)$$

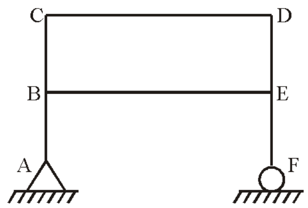
Similarly for space frame $D_S = R_e + 6m - 6j$

Due to presence of internal hinge in space frame

$$D_S = R_e + 6m - 6j - 3\Sigma(m' - 1)$$

Explanation :

(1) Frame without hinge : Let us take a frame as shown below



$$m = \text{number of members} = 6$$

Members are AB, BC, CD, DE, EF and BE.

$$R_e = \text{Number of support reactions} = 3$$

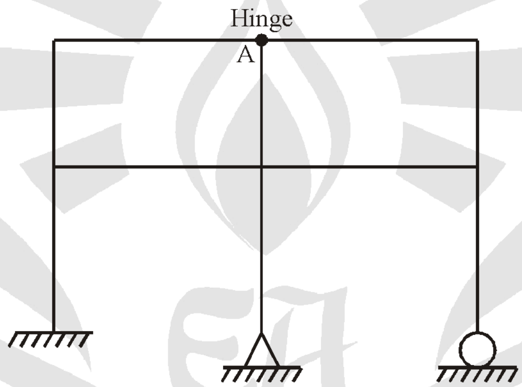
Reactions are R_{Ax}, R_{Ay} and R_{Fy}.

$$j = \text{Number of joints} = 6$$

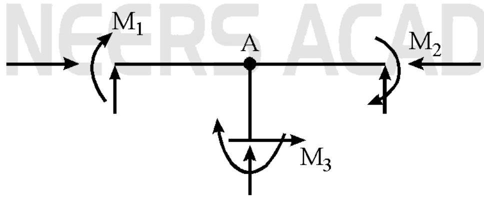
Joints are A, B, C, D, E and F.

$\therefore$
$$D_S = R_e + 3m - 3j$$
$$= 3 + 3 \times 6 - 3 \times 6 = 3$$

(2) Frame with hinges



Considering the above example, had the joint A been rigid, the degree of static indeterminacy would have been $R_e + 3m - 3j$. Due to the presence of joint with hinge, additional independent conditions available are as shown below



Equilibrium of moment at joint A i.e.

$$\Sigma M_A = 0$$

$$M_1 + M_2 + M_3 = 0 \quad ...(1)$$

But it is known that due to presence of hinge at A,

$$M_1 = 0 \quad ...(2)$$

$$M_2 = 0 \quad ...(3)$$

$$M_3 = 0 \quad ...(4)$$

Out of above three conditions (2), (3) and (4), only two are independent because third can be derived from equation (1). Hence number of independent additional equations are two i.e. (m′ – 1), where m′ = number of members meeting at hinge.

Thus, $D_S = R_e + 3m - 3j - (m' - 1)$

However if there are more than one hinge, we can have additional independent conditions as Σ(m′ – 1).

Thus, $D_S = R_e + 3m - 3j - \Sigma(m' - 1)$

Note that the hinges that we are considering are not the support hinges. They are member or joint hinges. On similar lines, it can be shown that for 3D-frame.

$D_S = R_e + 6m - 6j$ [Frame without hinge]

$D_S = R_e + 6m - 6j - 3\Sigma(m' - 1)$ [Frame with hinges]

1.7 Static Indeterminacy for Beams

Beam is made cantilever by adding constraint and removing all other support reactions.

D_S = Support removed – Constraint added

$= R_e - 3$

Where 3 is number of equation of equilibrium

and R_e = number of support reactions.

Degree of internal indeterminacy for beams is always zero. Hence

$D_S = D_{Se} + D_{Si}$

But $D_{Si} = 0$

∴ $D_S = D_{Se}$

1.8 Stability of Structure

Stability of structure is characterized into two parts

- External Stability
- Internal Stability

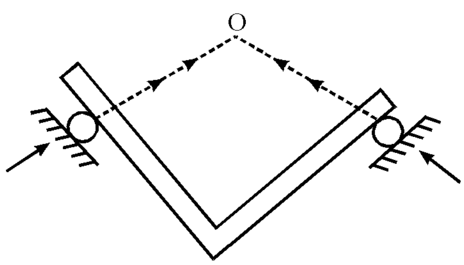
1.8.1 External Stability

If a body is sufficiently constrained by external reactions such that **rigid body movement** of structure can not be occurred, then the structure is said to be externally stable.

Necessary conditions for external stability are

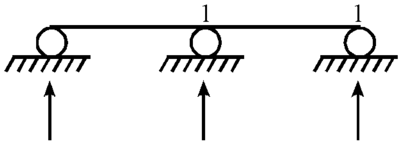
1. There should be minimum three external reactions.
2. Reactions should be
   (i) non-parallel and non-concurrent for plane structure
   (ii) non-parallel, non-concurrent and non-coplanar for space structure.
   (Concurrent means meeting at a single point).

Ex.:



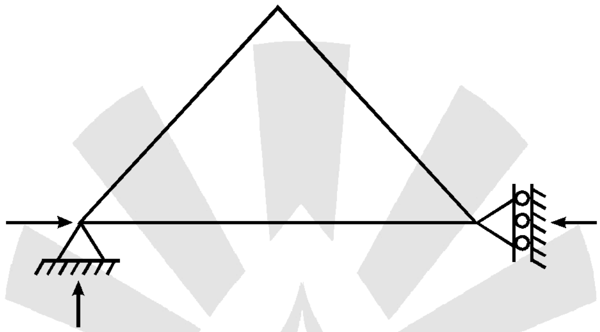
Reaction are concurrent, hence structure is externally unstable.

Ex.:



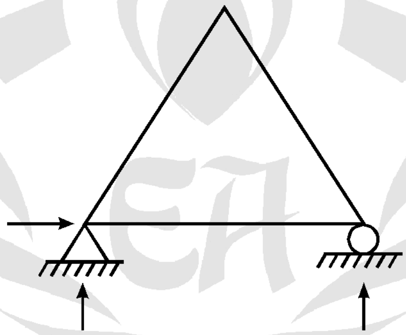
Reaction are parallel, hence inclined loading will lead to rigid body movement. So structure is externally unstable.

Ex.:



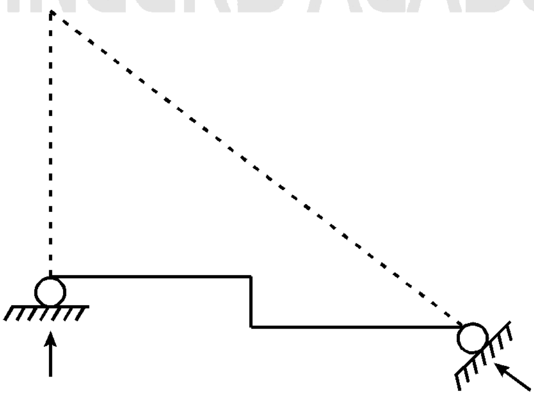
Reactions are concurrent, hence structure is externally unstable.

Ex.:



Three reactions are present that are non parallel and non-concurrent. Hence structure is externally stable.

Ex.:

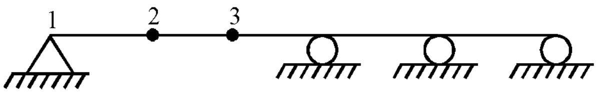


Reactions are concurrent, hence structure is externally unstable.

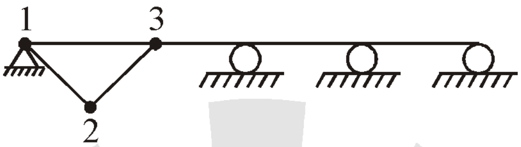
1.8.2 Internal Stability

When a part of the structure moves appreciably with respect to the other part, then the structure is said to be internally unstable.

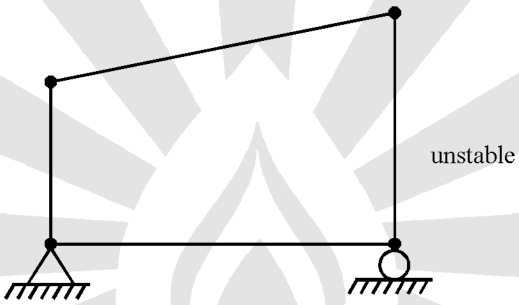
Ex.:



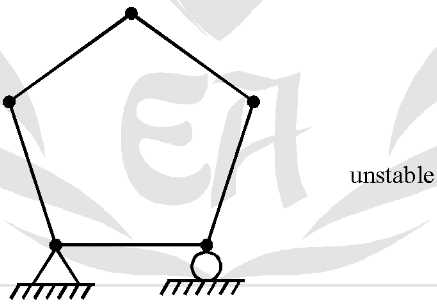
Three consecutive hinges are present that is a mechanism. The failure condition is shown below



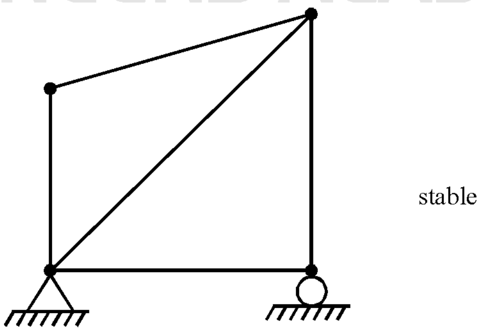
Ex.:



Ex.:

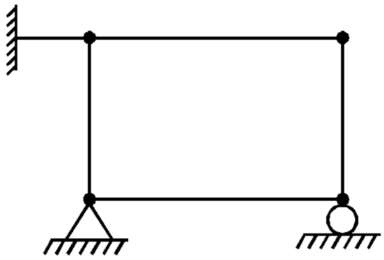


Ex.:



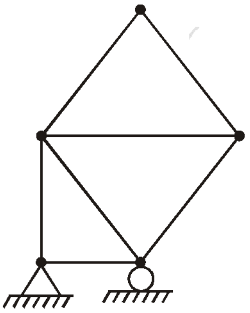
Note that application of a bracing has ensured that one part does not move appreciably with respect to the other part.

Ex.:

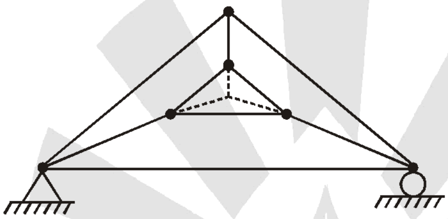


stable

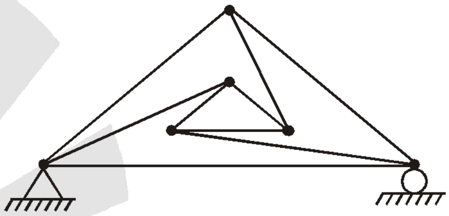
Fixing of one joint has ensured that appreciable deformation of one part cannot take place with respect to other.



stable



unstable



unstable

1.9 Static Indeterminacy of Trusses

A truss is designed in such a way that members of truss always carries only **axial forces.** Hence at truss joints equations of equilibrium available

$$\text{Number of available equilibrium equation} = \begin{cases} 2 & \text{for plane truss i.e.} \quad \Sigma F_x = 0, \Sigma F_y = 0 \\ 3 & \text{for space truss i.e.} \quad \Sigma F_x = 0, \Sigma F_y = 0, \Sigma F_z = 0 \end{cases}$$

Number of unknowns in truss is = $R_e + m$

Where, m = Number of members (because each member carry one force)

R_e = Number of reactions at supports.

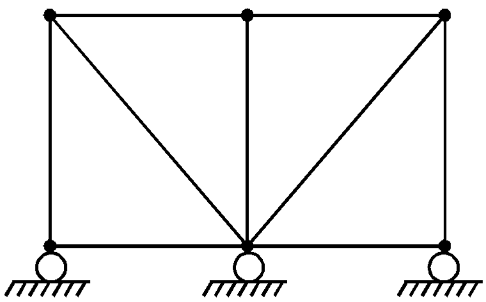
Hence $R_e + m - 2j = 0$ statically determinate plane-truss ...(1)

$R_e + m - 2j > 0$ statically indeterminate plane-truss ...(2)

$R_e + m - 2j < 0$ unstable truss ...(3)

However, condition (1) and (2) does not ensure that the truss will be stable. The stability should be checked visually or analytically.

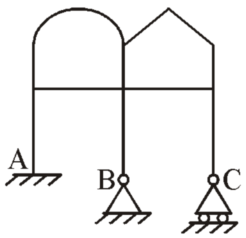
Ex.:



Externally unstable because of parallel support reactions. Hence determinate but unstable.

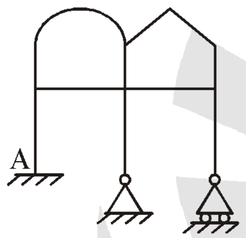
OBJECTIVE QUESTIONS

1. The static indeterminacy of the structure shown below



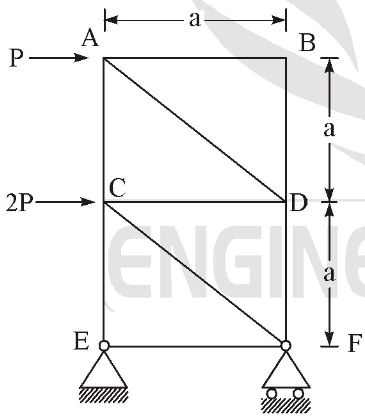
(a) 3 (b) 6
(c) 9 (d) 12

2. Determine the degree of freedom of the following frame



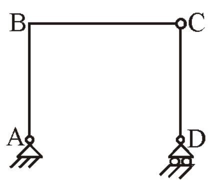
(a) 13 (b) 24
(c) 27 (d) 18

3. The force in the member 'CD' of the truss in fig is



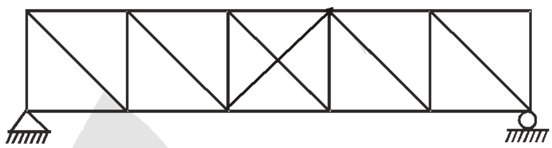
(a) Zero (b) 2P (Compression)
(c) P (Compression) (d) P (Tensile)

4. The plane frame shown below is :



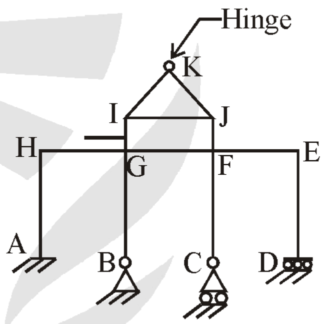
(a) statically indeterminate but unstable
(b) unstable
(c) determinate and stable
(d) none of the above

5. Determine static and kinematic indeterminacies for trusses



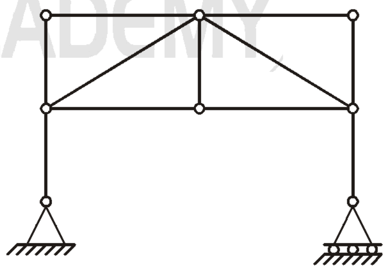
(a) 1, 21 (b) 2, 20
(c) 1, 22 (d) 2, 22

6. The static Indeterminacy of the structure shown below is.



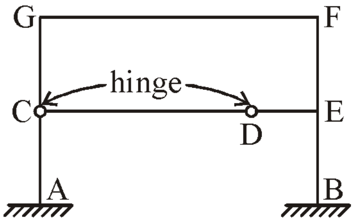
(a) 4 (b) 6
(c) 8 (d) 10

7. The structure shown below is



(a) externally indeterminate
(b) internally indeterminate
(c) determinate
(d) mechanism

8. The static Indeterminacy of the structure shown below is



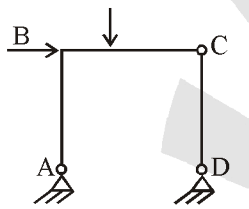
(a) unstable

(b) stable, determinate

(c) stable, indeterminate to 5th degree

(d) stable, indeterminate to 3rd degree

9. The plane figure shown below is



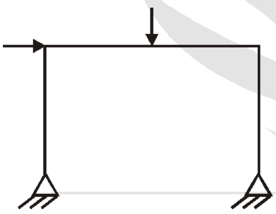
(a) Stable and statically determinate

(b) unstable and statically determinate

(c) stable and statically indeterminate

(d) unstable and statically indeterminate

10. The degrees of freedom of the following frames is.



(a) 3 (b) 4

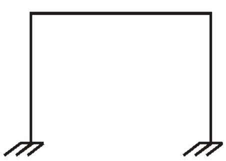
(c) 5 (d) 6

11. The kinematic indeterminacy of single bay portal frame fixed at the base is.

(a) One (b) Two

(c) Three (d) Zero

12. The kinematic indeterminacy of plane frame shown below is.



(a) 1 (b) 2

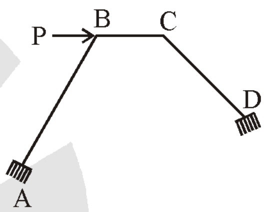
(c) 3 (d) zero

13. A beam fixed at the ends and subjected to lateral loads only is statically indeterminate and the degree of indeterminacy is

(a) One (b) Two

(c) Three (d) Four

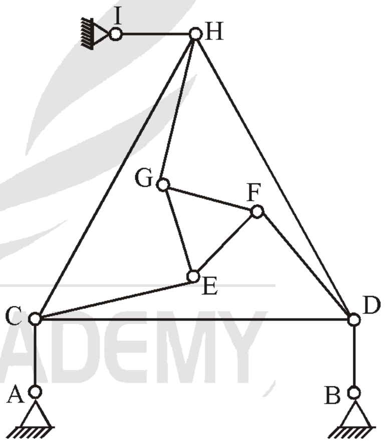
14. The degree of kinematic indeterminacy of the rigid frame with clamped ends at A and D shown in the figure is



(a) 4 (b) 3

(c) 2 (d) Zero

15. The following two statements are made with reference to the planar truss shown below:



I. The truss is statically determinate

II. The truss is kinematically determinate.

With reference to the above statements, which of the following applies?

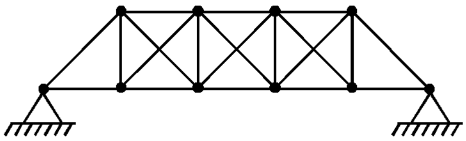
(a) Both statements are true

(b) Both statements are false

(c) II is true but I false

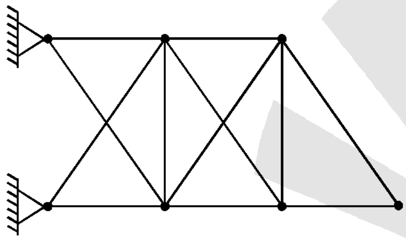
(d) I is true but II is false

16. The total degree of indeterminacy (both internal and external) for the bridge truss shown in the given figure is



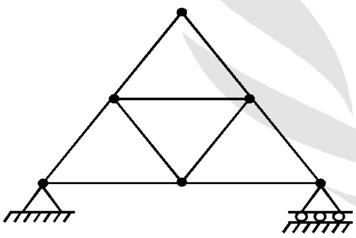
(a) 4 (b) 5
(c) 6 (d) 3

17. What is the degree of indeterminacy (both internal and external) of the cantilever plane truss shown in the figure below?



(a) 2 (b) 3
(c) 4 (d) 5

18. Consider the following statements with respect to the figure below of a typical articulated frame:

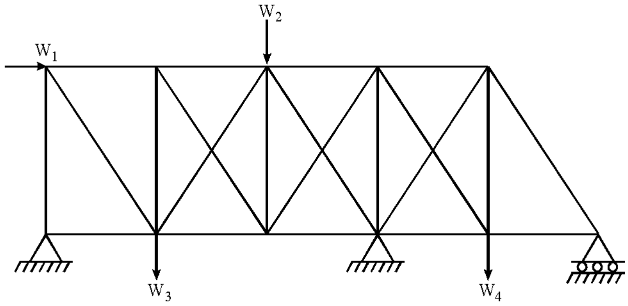


1. The frame is internally determinate and externally indeterminate.
2. The frame is internally indeterminate and externally determinate.
3. The frame is internally as well as externally determinate.
4. The frame is internally as well as externally indeterminate.

Which of these statements is/are correct?

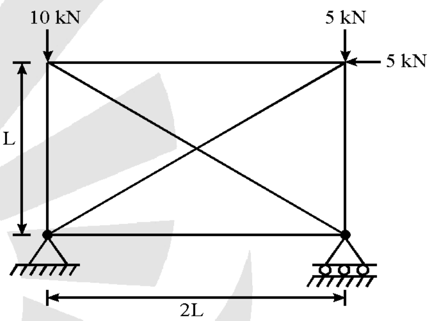
(a) 1 only (b) 1 and 2
(c) 3 only (d) 3 and 4

19. The degree of static indeterminacy of the pin-jointed plane frame shown in figure is



(a) 1 (b) 2
(c) 3 (d) 5

20. The frame shown below is redundant to



(a) single degree (b) two degree
(c) three degree (d) four degree

21. Match List-I (Type of structure) with List-II (Statical indeterminacy) and select the correct answer using the codes given below the lists

Number of member = m

Number of joints = n

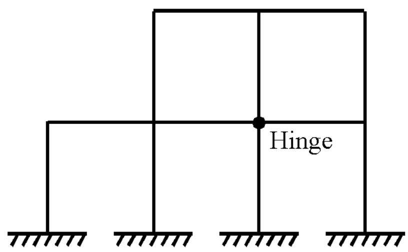
Number of external reaction elements = r

List-I	List-II
(A) Plane frame	1. m + r – 3n
(B) Space truss	2. 6m + r – 6n
(C) Space frame	3. 6m + r – 3n
	4. 3m + r – 3n

Codes :

	A	B	C
(a)	1	2	3
(b)	4	3	2
(c)	2	1	3
(d)	4	1	2

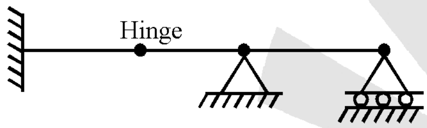
22. Total degree of indeterminacy (both internal and external) of the plane frame shown in the given figure is



(a) 10 (b) 11

(c) 12 (d) 15

23. The degree of indeterminacy of the beam given below is



(a) zero (b) one

(c) two (d) three

24. Which one of the following is true example of a statically determinate beam?

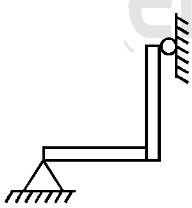
(a) One end is fixed and the other end is simply supported

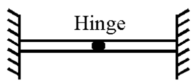
(b) Both the ends are fixed

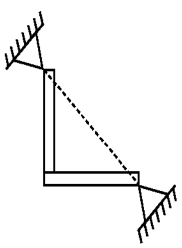
(c) The beam overhangs over two supports

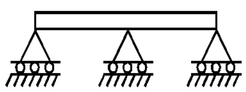
(d) The beam is supported on three supports

25. Which one of the following structures is statically determinate and stable?

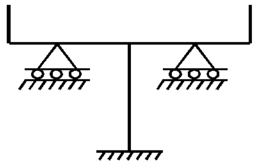
(a) 

(b) 

(c) 

(d) 

26. What is the degree of indeterminacy of the frame shown in the figure given below?



(a) 4 (b) 3

(c) 2 (d) zero

27. A determinate structure

(a) cannot be analyzed without the correct knowledge of modulus of elasticity

(b) must necessarily have roller support at one of its ends

(c) requires only statical equilibrium equations for its analysis

(d) will have zero deflection at its ends

28. A statically indeterminate structure is the one which

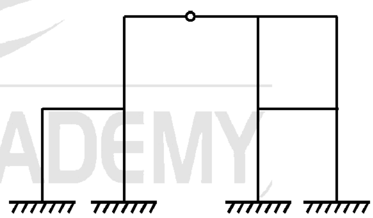
(a) cannot be analyzed at all

(b) can be analyzed using equations of statics only

(c) can be analyzed using equations of statics and compatibility equations

(d) can be analyzed using equations of compatibility only

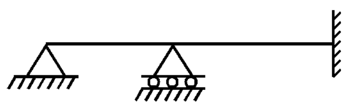
29. What is the statical indeterminacy for the frame shown below?



(a) 12 (b) 15

(c) 11 (d) 14

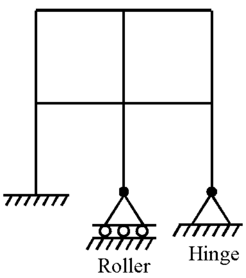
30. What is the number of independent degrees of freedom of the two-span continuous beam of uniform section shown in the figure below?



(a) 1 (b) 2

(c) 3 (d) 4

31. What is the kinematic indeterminacy for the shown below? (members are inextensible)



(a) 6 (b) 11

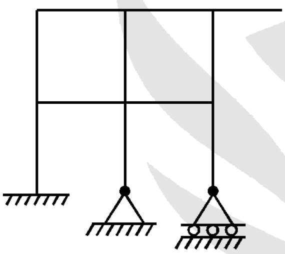
(c) 12 (d) 21

32. If the axial deformation is neglected, what is the kinematic indeterminacy of a single bay portal frame fixed at base?

(a) 2 (b) 3

(c) 4 (d) 6

33. For the plane frame with an overhang as shown below, assuming negligible axial deformation the degree of static indeterminacy 'd' and the degree of kinematic indeterminacy 'k' are



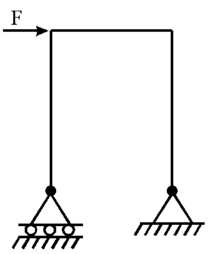
(a) d = 3 and k = 10

(b) d = 3 and k = 13

(c) d = 9 and k = 10

(d) d = 9 and k = 13

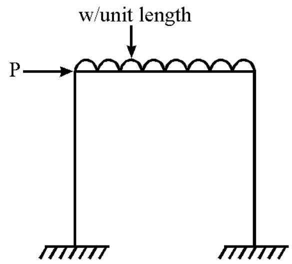
34. Considering beam as axially rigid, the degree of freedom of a plane frame shown below is



(a) 9 (b) 8

(c) 7 (d) 6

35. The frame shown in the given figure has



(a) one unknown reaction component

(b) two unknown reaction components

(c) three unknown reaction components

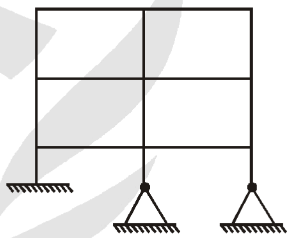
(d) six unknown reaction components

36. A perfect plane frame having n number of members and j number of joints should satisfy the relation

(a) $n < (2j - 3)$ (b) $n = (2j - 3)$

(c) $n > (2j - 3)$ (d) $n = (3 - 2j)$

37. The total (both internal and external) degree of static indeterminacy of the plane frame shown in the given figure is



(a) 18 (b) 16

(c) 14 (d) 13

38. Statical indeterminacy for 2D truss is

(a) m + r – 2 j (b) m + r – 3 j

(c) m + j – 2 r (d) m – j + 2 j

39. Statical indeterminacy for 3D truss is

(a) m + r – 3 j (b) m + r – 2 j

(c) m + 3 j – r (d) m + j – 3 r

40. Statical indeterminacy for 2D beams & frames is

(a) 2(m – j) + r – f (b) 3(m – j) + r – f

(c) 3(m – j) + f – r (d) 3(m – r) + j – f

41. Statical indeterminacy for 3D beams & truss is

(a) 3(m – j) + r – f (b) m + r – 3 j

(c) 6(m – j) + r – f (d) 6(m – r) + j – f