Conventional Practice Book

RPSC-AEn

Rajasthan Public Service Commission-Assistant Engineer

Mains Examination

STRENGTH OF MATERIALS

HIGHLIGHTS

- As per RPSC-AEn Mains Syllabus.
- Coverage of **2 Marks, 5 Marks and 20 Marks** Questions with Explanations.
- Previous Year Questions with Detailed Solutions.

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CHAPTER 1

STRENGTH OF MATERIALS

Conventional Solved Question and Detailed Solutions

2 MARKS QUESTIONS

1. What do you understand by proof resilience?

Solution :

Proof Resilience

Resilience is the ability of a material to absorb energy when it is deformed elastically and release that energy upon unloading proof resilience is defined as the maximum energy that can be absorbed up to the elastic limit without creating a permanent distortion.

$$\text{Modulus of Resilience} = \frac{\text{Proof Resilience}}{\text{Volume}} = \frac{\sigma_{EL}^2}{2E}$$

σ_{EL} = Strain at elastic limit

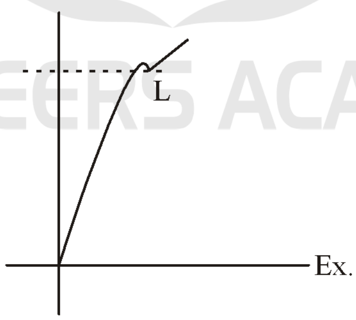
E = Modulus of elasticity

2. Define yield point.

Solution :

Yield Point

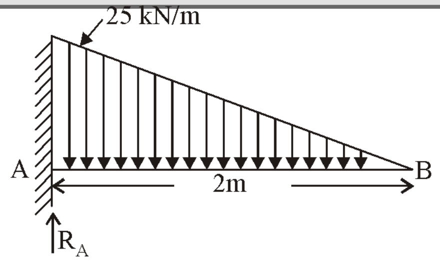
A yield point is the point at which a material begins to flow or change shape permanently. The yield strength is calculate corresponding to this point.



3. A cantilever 2 m long carries a gradually varying load over the whole span, from zero at free end and increment of load 25 kN/m towards the fixed end. Determine the value of shear force at the fixed end.

Solution :

As we know



$$\Sigma F_y = 0$$

$$R_A = \frac{1}{2} \times 2 \times 25$$

$$R_A = 25 \text{ kN}$$

Shear force at fixed end = 25 kN

4. Explain "Point of contraflexure"

***Solution* :**

Point of contraflexure

Point of contraflexure is the point where bending moment changes its sign i.e. from positive value to a negative value or vice versa at point of contraflexure the value of bending moment is zero.

5. What is principle of virtual work?

***Solution* :**

Principle of virtual work

"The Principle of virtual work is related with equilibrium of bodies and may be states as follows :

"If a system in equilibrium under the action of a set of forces is given a virtual displacement the virtual work done by forces will be zero"

6. Define Poisson's ratio of materials.

***Solution* :**

Poisson's Ratio :

The ratio of lateral strain to the longitudinal strain is a constant for given material.

When the material is stressed with in the elastic limit. This ratio is called Poisson ratio.

$$\text{Poisson's Ratio}(\mu) = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

Its varies from 0. to 0.5

$\mu = 0$ for cork

$\mu = 0.5$ for perfectly plastic body

$\mu = 0.25$ to 0.42 for elastic metals

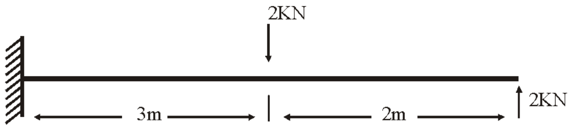
$\mu = 0.1$ to 0.2 for concrete

$\mu = 0.286$ mild steel

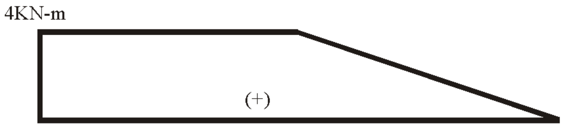
It is greater for ductile metals than for brittle metals.

11. A cantilever AB, 5m long is fixed at end A. At end B, an upward load of 2kN is applied. At point C, 2m from end B, a downward load of 2kN is applied. Draw the bending moment diagram of the cantilever.

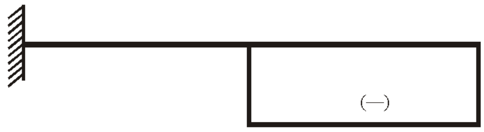
Solution:



Loading Diagram



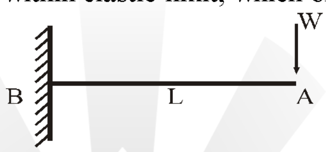
BMD



SFD

12. If two candilever beams of identical dimensions but made of mild steel and gray cast iron are subjected to same plint load at the free end, within elastic limit, which one will deflect more and why?

Solution: $E_S = 200GPa$



$E_{C.I} = 10GPa$

Maximum Defliction

$$Y_A = \frac{WL^3}{3EI} \quad , \quad Y_A \propto \frac{1}{E} \quad ; \qquad \because [E_S > E_{C.I}]$$

Cast Iron beam deflects more than mild steel beam, it is because cast Iron has less modulus of elasticity compared to mild steel.

13. Differentiate between strut and column. What is the genral expression used for determination of their critical load.

Solution:

Strut	Column
(1) It is also subjected to axial compressive load but can be inclined also.	(1) It is a vertical structural member which is always subjected to axial compressive load.
(2) If it is too long, then fails by buckling	(2) If it is too long, then fails by Buckling
(3) It can be used to transfer moderate loads	(3) It can be used to transfer heavy loads.

<u>Expression used for determaintion of critical load</u> :

$$\left[\quad P = \frac{\pi^2 EI}{L^2} \quad \right]$$

Where E is the young;s modulus of the column material, I is the area moment of Inertia of the cross section and L in the Length of the column.

14. What is the value of Euler's bucking load for an axially loaded pin-ended (hinged at both ends) strut of length 'I' and flexural rigidity 'EI'? What would be the order of Euler's buckling load carrying capacityh of a similar strut but fixed at both ends in terms of the load carrying capacity of the earlier one?

Solution: Euler's Buckling Load

$$P_e = \frac{\pi^2 EI}{L^2}$$

Where , E $\rightarrow$ young's Modulus

I $\rightarrow$ Moment of tnestia

L $\rightarrow$ Length of column

F-F $\rightarrow$ Fixed at both ends

H-H $\rightarrow$ Hinged at both ends

$$\frac{P_{e(F-F)}}{P_{e(H-H)}} = \frac{L^2_{H-H}}{L^2_{F-F}} \quad \frac{L^2}{\left(\frac{L}{2}\right)^2} = 4$$

15. Two long columns are made of indentical lengths 'L' and flexural rigidities 'EI'. Column 1 is hinged at both ends whereas for column 2 one end is fixed and the other end if free.

(i) Write the expression for Euler's buckling load for column 1.

(ii) What is the ratio of Euler's buckling load of column 1 to that of column 2?

Solution: (i) $P_e = \frac{\pi^2 EI}{L^2}$

(ii) $$\frac{P_{(H-H)}}{P_{(Fix-free)}} = \frac{(L_{fix-free})^2}{(L_{H-H})^2} = \frac{(2L)^2}{L^2} = 4$$

16. How stress differs from strength of materials?

Solution :

17. What is the coefficient of thermal expansion.

Solution :

5 MARKS QUESTIONS

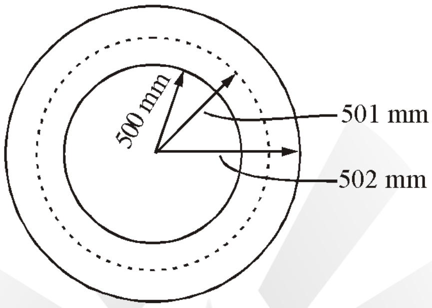
27. A wire of 2 mm diameter is wound round a circular shaft of 1 m diameter. What are the maximum normal and shear stresses in the wire? Specify the planes of their occurrence. E = 1 × 10^5 N/mm^2. Assume no applied tensile pull on the wire.

Solution:

Diameter of wire, d = 2 mm

Diameter of shaft, D =1000 mm

Modulus of Elasticity, E = 1 × 10^5 N/mm^2



$\because$ $$\frac{\sigma}{y} = \frac{M}{I} = \frac{E}{R}$$

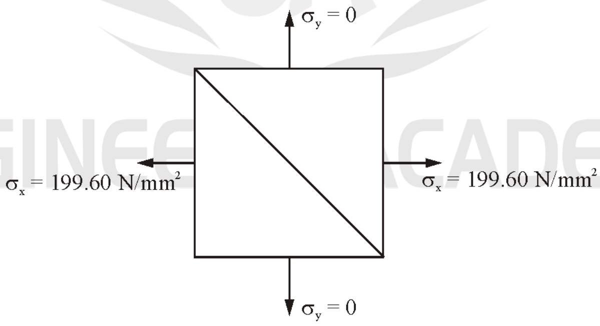
$\Rightarrow$ $$\sigma = \frac{Ey}{R}$$

for wire $$y = \frac{d}{2} = \frac{2}{2} = 1\text{mm}$$

R = control radius = 501 mm

$$\sigma = \frac{1 \times 10^5 \times 1}{501} = 199.6 \text{ N/mm}^2$$

Maximum shear stress will be at a plane 45° to the plane of maximum normal stress.



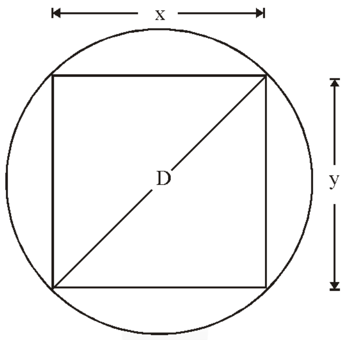
$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{199.60 - 0}{2} \quad (\sigma_1 = \sigma_x, \sigma_2 = 0)$$

$$= 99.80 \text{ N/mm}^2$$

28. A circular log of timber has diameter D. Determine the dimension of the strongest rectangular section, one can cut from this log.

Solution:

Let the dimension of the strongest section cut from the circular log of dia D be x and y.



Hence $D^2 = x^2 + y^2$ ...(1)

The strength of any section will depend on its flexural strength i.e. Z

Flexural srength of rectangular section is given be

$$\Rightarrow \quad Z = \frac{I}{y} = \frac{\frac{xy^3}{12}}{\frac{y}{2}} = \frac{xy^2}{6}$$

for Z to be maximum differentiating it with either x or y

$$\Rightarrow \quad \frac{dZ}{dx} = 0$$

$$\Rightarrow \quad \frac{d}{dx}\left(\frac{xy^2}{6}\right) = 0$$

From (1); $y^2 = D^2 - x^2$

$$\Rightarrow \quad \frac{d}{dx}\left\{\frac{x}{6} \times \left(D^2 - x^2\right)\right\} = 0$$

$$\Rightarrow \quad \frac{1}{6}\frac{d}{dx}\left(D^2x - x^3\right) = 0$$

$$\Rightarrow \quad \frac{1}{6}\left(D^2 - 3x^2\right) = 0$$

$$\Rightarrow \quad x^2 = \frac{D^2}{3}$$

or $$x = \frac{D}{\sqrt{3}}$$

Put value of x in equation (1)

$$D^2 = \frac{D^2}{3} + y^2$$

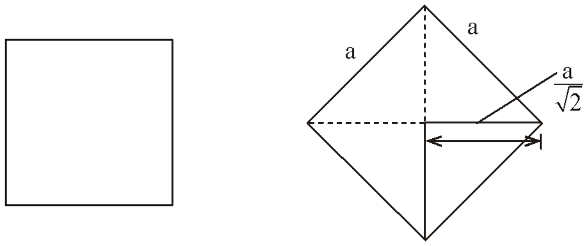
$$\Rightarrow \quad y = \sqrt{\frac{2}{3}}D$$

Dimension of strongest rectangular section from circular log or dia D will be

$$x = \frac{D}{\sqrt{3}} \text{ and } y = \sqrt{\frac{2}{3}}D$$

29. A beam of square section of the side 'a' is placed such that (i) two sides are horizontal (ii) one diagonal is horizontal. Find the ratio of the moments of resistance of the section in two positions for same permissible bending stress.

Solution:



(i) (ii)

Moment of Resistance (MOR) = σ_{max} Z

$$Z_1 = \frac{I_1}{a/2} = \frac{\frac{a^4}{12}}{a/2} = \frac{a^3}{6}$$

$$Z_2 = \frac{I_1}{a/\sqrt{2}} = \frac{2(a\sqrt{2})(a/\sqrt{2})^3}{12(a/\sqrt{2})} = \frac{2\sqrt{2}a^3}{12} = \frac{a^3}{6\sqrt{2}}$$

$$\frac{MOR_1}{MOR_2} = \frac{f_{max}Z_1}{f_{max}Z_2} = \frac{Z_1}{Z_2} = \frac{a^3/6}{a^3/6\sqrt{2}} = \sqrt{2}$$

30. A vertical cylindrical steel storage tank has 30 m diameter and the same is filled upto a depth of 15 m with the gasoline of relative density 0.74. If the yield stress for steel is 250 MPa, find the thickness required for the wall plate. Adopt a factor of safety of 2.5 and neglect localized bending effects, if any.

Solution:

Dia of cylinder = 30m, depth of cylinder = 15m

Hoop stress $(\sigma_n) = \frac{pd}{2t}$

where p = pressure = ρgh

$= (0.74 \times 1000) \times 9.81 \times 15$

$= 108891\ N/m^2 = 0.108891\ N/mm^2$

yield stress in steel = 250 MPa

Factor of safety = 2.5

(σ) permissible stress in steel = 100 N/mm^2

For safety $\sigma_n \leq \sigma$

$$\frac{0.108891 \times 30000}{2t} \leq 100$$

$\Rightarrow$ 16.33 mm $\leq$ t

longitudinal stress $(\sigma_l) = \frac{pd}{4t} \leq 100$

$$\Rightarrow \frac{0.108991 \times 30000}{4t} \leq 100$$

$\Rightarrow$ t $\geq$ 8.165 mm

So we adopt t = 16.33 mm.

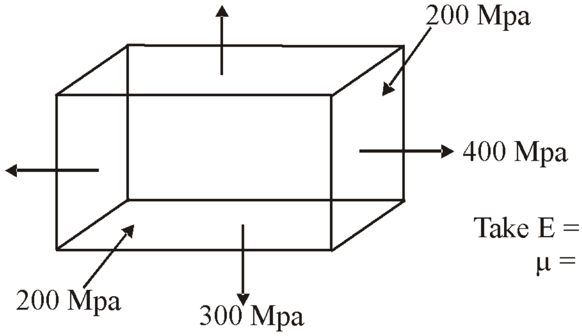
39. A compound bar consists of a circular rod of steel of diameter 25 mm, rigidly fitted into a copper tube of internal diameter 25 mm and thickness 2.5 mm. If the bar is subjected to an axial load of 100 kN, find the stresses developed in the two materials. Given, $E_s = 2 \times 10^5$ N/mm^2 and $E_c = 1.2 \times 10^5$ N/mm^2.

Solution:

40. A steel bar 40 mm × 40 mm, 300 cm long is subjected to an axial pull of 12.8 tonnes. Taking $E = 2 \times 10^6$ kg/cm^2 and possion's ratio as 0.3 calculate the alterations in the length and sides of the bar during the extension.

Solution :

41. For the element loaded as shown in figure calculate volumetric strain.



Take $E = 2 \times 10^5\ N/mm^2$

$\mu = 0.25$

Solution :

50. A Steel tube 2.5 cm external diameter and 1.8 cm internal diameter enclose a copper rod 1.6 cm diameter to which it is rigidly joined at each end. If, at a temperature of 20°C there is no longitudinal stress, calculate the stresses in the rod and tube when the temperature is raised to 210°C.

Given : E^s=210GPa and $\alpha_s = 12\times10^{-6}/°C$

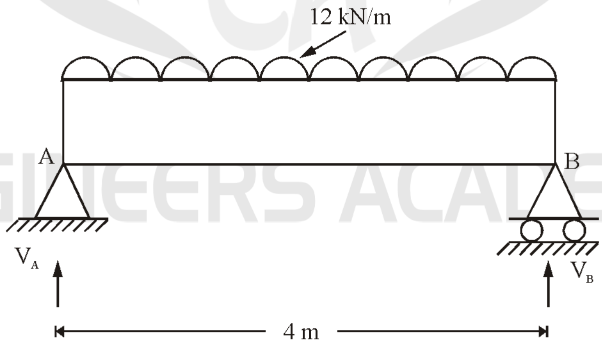
E_s = 100GPa and $\alpha_c = 20\times10^{-6}/°C$

Solution:

20 MARKS QUESTIONS

51. A simply supported beam is 4 m long and carries a UDL of 12 kN/m over its entire length. The cross-section of the beam is 300mm width × 400mm depth. Find the principal stresses and their directions at a point 100 mm below the neutral axis and at 1/4 of the span.

Solution:



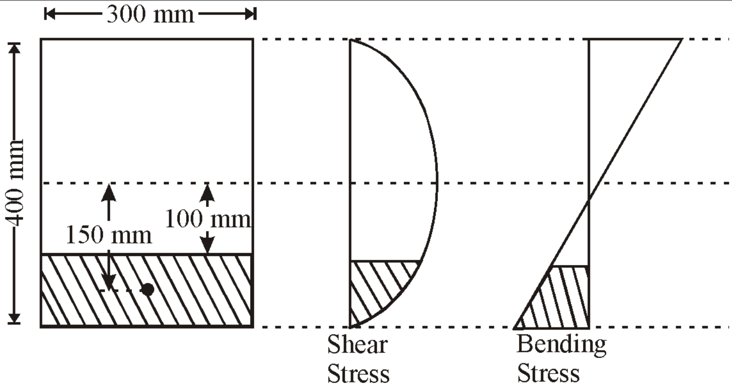
$$\Rightarrow \qquad \Sigma F_y = 0$$

$$\Rightarrow \qquad V_A + V_B = 12 \times 4 = 48$$

Since the loading is symmetrical, hence reactions will be equal.

$$V_A = 24 \text{ kN}$$

$$V_B = 24 \text{ kN}$$



BM at $\frac{1}{4}$ of span from left support i.e. 1 m from left support is given as

$$M = 24 \times 1 - 12 \times 0.5 \times 1$$

$$\Rightarrow \quad M = 24 - 6 = 18\text{kN-m}$$

Shear force at 1m from left support is given as

$$S = 24 - 12 \times 1 = 12 \text{ kN}$$

Now, bending stress at the level of P (100 mm below NA) will be

$$\sigma = \frac{M}{I}y = \frac{18 \times 10^6 \times 12 \times 100}{300 \times 400^3}$$

$$\sigma = 1.125 \text{ N/mm}^2 \text{ (tensile)}$$

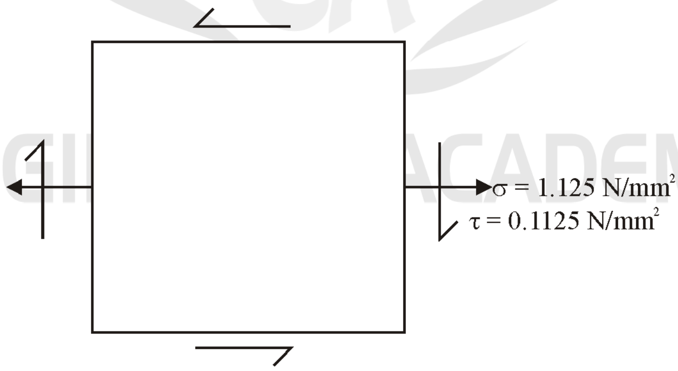
and shear stress τ at the level of P (100 mm below NA) will be

$$\tau = \frac{SA\overline{Y}}{Ib}$$

$$\tau = \frac{12 \times 10^3 \times 100 \times 300 \times 150 \times 12}{300 \times 400^3 \times 300}$$

$$\tau = 0.1125 \text{ N/mm}^2$$

Constructing stress element



$$\sigma_x = 1.125 \text{ N/mm}^2$$

$$\sigma_y = 0$$

$$\tau_{xy} = -0.1125 \text{ N/mm}^2$$

The principal stresses is given as

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

$\Rightarrow \quad \sigma_{1,2} = \dfrac{1.125+0}{2} \pm \sqrt{\left(\dfrac{1.125-0}{2}\right)^2 + (-0.1125)^2}$

$\therefore \quad \sigma_1 = 1.136 \text{ N/mm}^2 \text{ and } \sigma_2 = 0.011 \text{N/mm}^2$

and $\quad \tan 2\theta_P = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y} = \dfrac{-2 \times 0.1125}{1.125 - 0}$

$\Rightarrow \quad \theta_{P_1} = -5.65^\circ$

$\Rightarrow \quad \theta_{P_2} = -95.65^\circ$

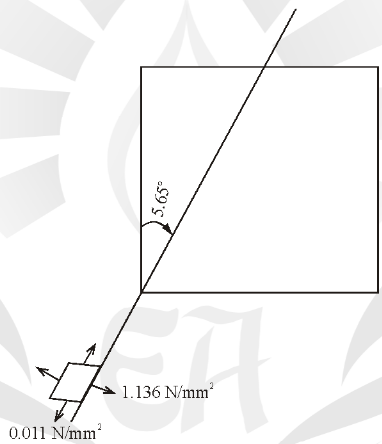
Check :

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

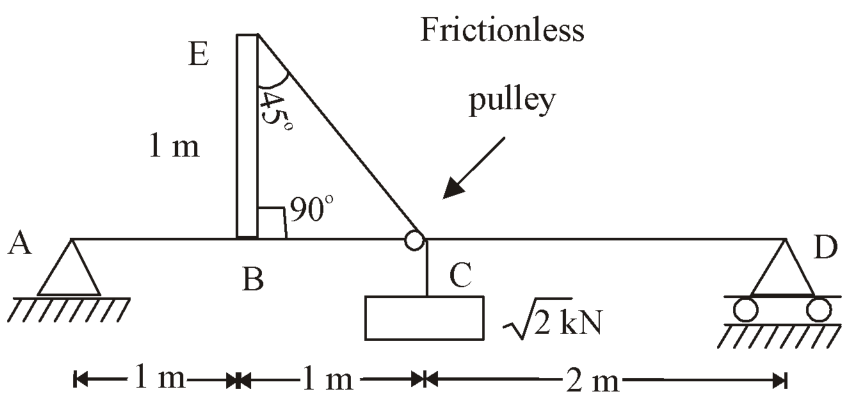
$$= \frac{1.125+0}{2} + \frac{1.125-0}{2} \cos(2 \times (-5.65^\circ)) - 0.1125 \sin(2 \times (-5.65^\circ))$$

$$= 0.5625 + 0.5516 + 0.0220 = 1.136 \text{ Nmm}^2$$

Hence major principal plane is 5.65° clockwise and minor principal plane is 95.65° clockwise from the vertical.

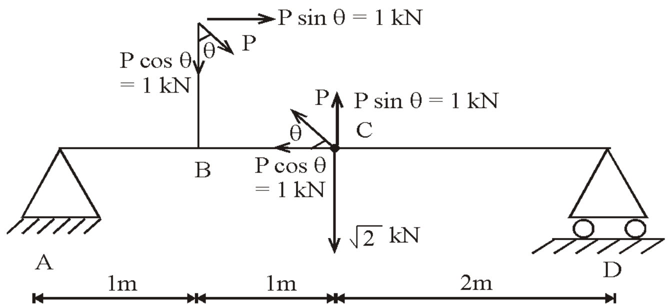


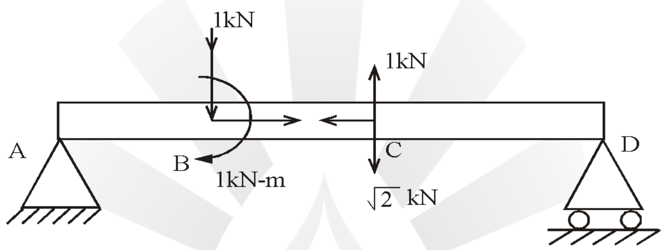
52. A beam ABCD is hinged at A and simply supported at D.A vertical strut BE, 1m long is fixed at B, 1m from A and a frictionless pulley is attached to the beam at C, 2 m from A. A flexible string carries a load $\sqrt{2}$ kN and passes over the pulley and is attached to the strut at E as shown in the figure. Draw the BM, SF and axial force diagrams of the beam.



Solution:

Draw FBD by removing string between E and C.





$$\Sigma M_A = 0$$

$$\Rightarrow \quad R_D \times 4 = \left[\left(\sqrt{2}-1\right)\times 2\right] + (1\times 1) + 1$$

$$\Rightarrow \quad 4R_D = 2\sqrt{2} - 2 + 1 + 1$$

$$\Rightarrow \quad R_D = \frac{1}{\sqrt{2}} = 0.707\text{kN}$$

$$\Sigma F_y = 0$$

$$\Rightarrow \quad R_A + R_D = \sqrt{2}$$

$$\Rightarrow \quad R_A = \sqrt{2} - \frac{1}{\sqrt{2}} = 0.707 \text{ kN}$$

SFD :

Portion AB :

$V = 0.707$ kN constant between A and B

Portion BC :

$V = R_A - 1$

$= -\ 0.293$ kN constant between B to left of C

Portion CD :

$V = R_A - 1 - \sqrt{2} + 1$

$V = 0.707 - \sqrt{2}$

$= -\ 0.707$ kN constant Right of C to D.

For BMD :

Portion AB : (x from A)

$$M_x = R_A x$$
$$= 0.707x$$

at x = 0 $\quad M_A = 0$

at x = 1 $\quad M_B = 0.707 \times 1 = 0.707$ kN–m

Portion BC : (x from A)

$$M_x = R_A x + 1 - 1(x-1)$$
$$= 0.707x + 1 - (x-1) = -0.293x + 2$$

at x = 1 $\quad M_B = 1.707$ kN–m

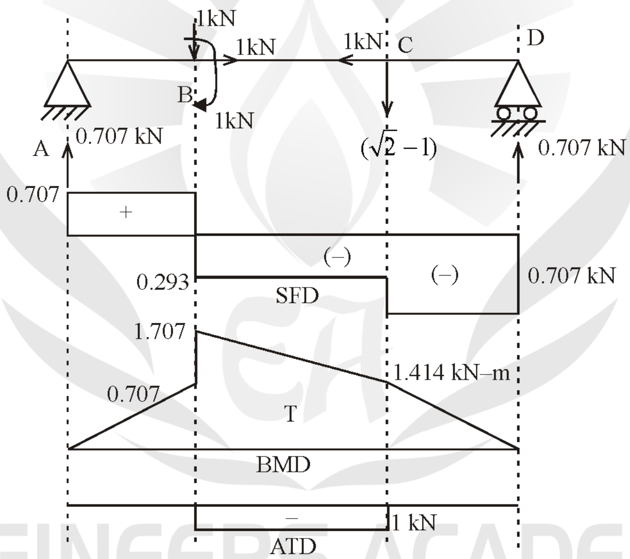
at x = 2 $\quad M_C = -0.293 \times 2 + 2 = 1.414$ kN–m

Portion CD : (x from A)

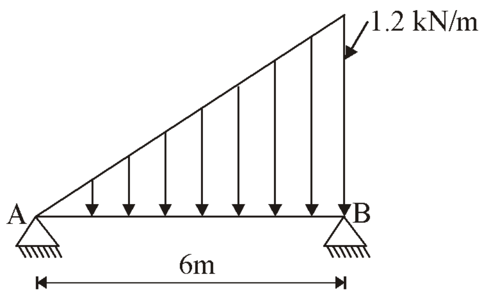
$$M_x = R_A x + 1 - 1(x-1) - (\sqrt{2}-1)(x-2)$$
$$= 0.707x + 1 - (x-1) - (\sqrt{2}-1)(x-2)$$

at x = 2 $\quad M_C = 1.414$

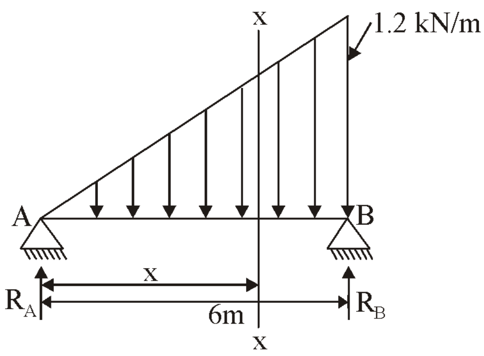
at x = 4 $\quad M_D = 0$



53. A beam AB of 6m span carries a distributed load of varying intensity as shown in the figure below. Calculate the deflection at the centre of the beam. Load intensity at support B = 1.2 kN/m, EI = 420×10^9 N-mm^2.



Solution:



Support Reactions :

$$R_A + R_B = \frac{1}{2}\times 6\times 1.2$$

$$R_A + R_B = 3.6 \text{ kN}$$

$$\Sigma M_B = 0$$

$$R_A \times 6 - \frac{1}{2} \times 1.2 \times 6 \times \frac{1}{3} \times 6 = 0$$

$$R_A = 1.2 \text{ kN}$$

$$R_B = 2.4 \text{ kN}$$

Using Macaulay method

6m $\rightarrow$ 1.2 kN/m

For X $\rightarrow$? $= \frac{1.2x}{6} = 0.2x$

$$M_x = 1.2x - \frac{1}{2}\times 0.2x \times x \times \frac{1}{3} x$$

$$M_x = 1.2x - \frac{x^3}{30}$$

$$EI\frac{d^2y}{dx^2} = M_x$$

$$EI\frac{dy}{dx} = \frac{1.2x^2}{2} - \frac{x^4}{30\times 4} + C_1$$

$$EIy = \frac{1.2x^3}{6} - \frac{x^5}{4\times 5\times 30} + C_1x + C_2$$

At A, ; x = 0 ; y = 0

$$C_2 = 0$$

At B, x = 6, y = 0

$$0 = \frac{1.2\times 6^3}{6} - \frac{6^5}{4\times 5\times 30} + C_1(6)$$

$$C_1 = -5.04$$

Deflection at mid span

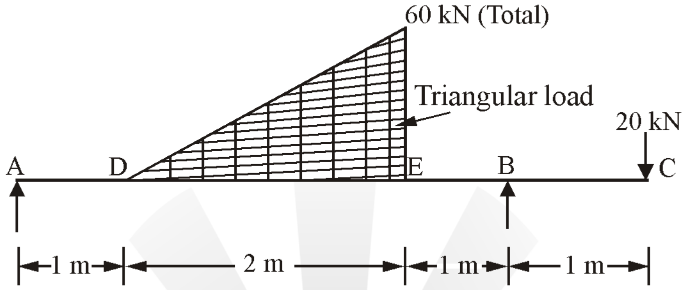
$$EIy_{mid} = \frac{1.2\times3^3}{6} - \frac{3^5}{4\times5\times30} - 5.04\times3$$

$$y_{mid} = \frac{-10.125}{420\times10^9\times10^{-3}\times10^{-6}}$$

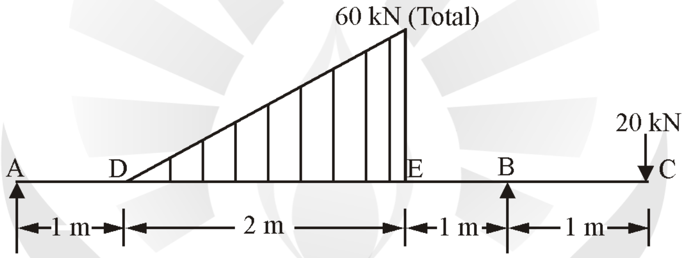
$$y_{mid} = -24.11 \text{ mm (downward)}$$

Deflection at centre $= 24.11$ mm ($\downarrow$)

54. Draw the bending moment and shear force diagrams for overhanging beam shown in the figure below. Indicate the significant values including the point of contraflexure.



Solution:



Sign Convention

SF : $\uparrow\downarrow$ Positive

BM : Sagging $\smile$ + ve

Let R_A and R_B be the support reaction at A and B respectively.

Sum of vertical force in y direction $\Sigma F_y = 0$

$$\Rightarrow \quad R_A + R_B = 80 \qquad \text{...(1)}$$

$$\Sigma M_A = 0$$

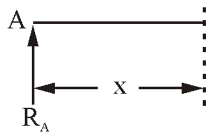
$$20\times5 - R_B\times4 + \frac{1}{2}\times2\times60\times\left(1+2\times\frac{2}{3}\right) = 0$$

$$\Rightarrow \quad 4R_B = 100 + 60\left(\frac{7}{3}\right) = 60 \text{ kN}$$

$$\Rightarrow \quad R_A = 80 - 60 = 20 \text{ kN}$$

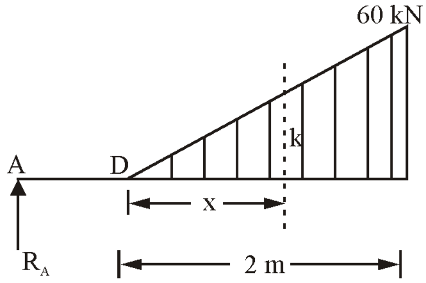
Shear Force Diagram :

Section AD : Consider a section at a distance x from A between AD



S.F. $= + R_A = + 20$ kN (constant from A to D)

Section DE : Consider a section at a distance x from D.



From similar triangle property

$\Rightarrow$ $$\frac{60}{2} = \frac{K}{x}$$

$\Rightarrow$ $$K = 30\,x$$

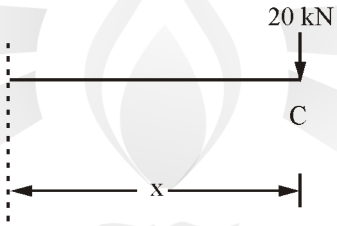
$$S.F. = +20 - \frac{1}{2} \times x \times 30x = 20 - 15x^2$$

at point D (x = 0) $\quad$ S.F. = + 20 kN

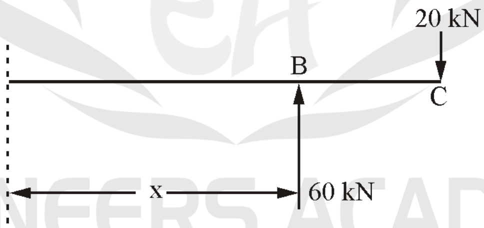
at point E (x = 2 m) $\quad$ S.F. = + 20 − 15 × 2^2 = − 40 kN

Section CB : Consider a section at a distance x from C

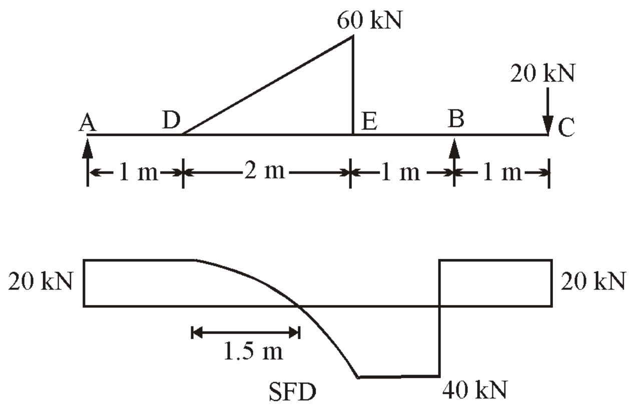
S.F. = + 20 kN (constant from C to B).



Section BE : Consider a section at a distance x from B



S.F. = 20 − 60 = − 40 kN (Constant from E to B)



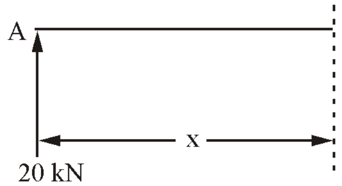
In Section DE, value of x at which S.F. will be zero is

$$S.F. = 20 - 15\,x^2 = 0$$

$$\Rightarrow \qquad x = \frac{2}{\sqrt{3}} = 1.15 \text{ m}$$

Bending Moment Diagram :

Section AD : Consider a section at a distance x from A.

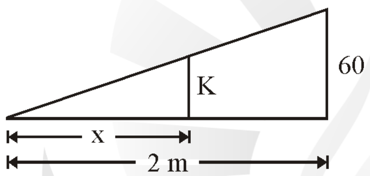


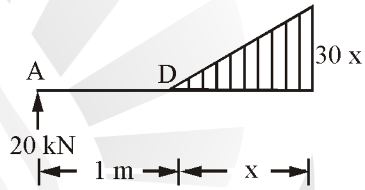
$$BM = 20x \text{ (Sagging)}$$

at point A (x = 0) $\quad BM = 0$

at point D (x = 1 m) $\quad BM = 20 \times 1 = 20$ kN-m

Section DE : Consider a section at a distance x from D.





From similar triangle property

$$\Rightarrow \qquad \frac{60}{2} = \frac{K}{x}$$

or $\quad K = 30\,x$

$$\Rightarrow \qquad BM = 20\,(1 + x) - \frac{1}{2} \times (30x) \times x \times \frac{x}{3}$$

$$= 20 + 20\,x - 5x^3$$

at point D (x = 0) $\quad BM = 20$ kN–m

at point E (x = 2 m) $\quad BM = 20 + 20 \times 2 - 5 \times 2^3 = 20$ kN–m

Since BM at two extreme points of section DE is equal hence there must be a point having zero slope and having maximum or minimum value of BM within the section

At point of maximum and minimum BM:

$$\frac{d}{dx}(BM) = 0$$

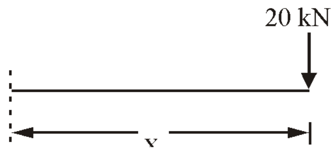
$$\Rightarrow \qquad \frac{d}{dx}(BM) = \frac{d}{dx}(20 + 20x - 5x^3) = 20 - 15x^2 = 0$$

$$\Rightarrow \qquad x = 1.15 \text{ m}$$

$\frac{d^2(BM)}{dx^2}$ is having negative value, hence the BM will be maximum.

and maximum value of $\quad BM = 20 + 20 \times 1.15 - 5 \times 1.15^3 = 35.39$ kN–m

Section CB : Consider any section at a distance x from C.

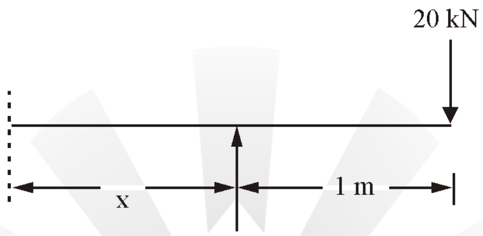


$\Rightarrow$ BM = – 20x (Hogging)

at point C (x = 0) BM = 0

at point B (x = 1 m) BM = – 20 kN–m

Section BE : Consider any section at a distance x from B.



$\Rightarrow$ BM = – 20(1 + x) + 60 x

at point B (x = 0) BM = – 20 kN–m

at point E (x = 1 m) BM = – 20 (1 + 1) + 60 × 1= 20 kN–m

In between section BE a point is having zero BM, called point of contraflexure because BM changes its sign at this point.

$\Rightarrow$ $-20(1 + x) + 60x = 0$

or $x = 0.5$